Nuclear matrix elements of double-beta decay by QRPA and attempt to extend RPA

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1. Nuclear matrix elements of $\beta\beta$ decay – originality of my calculation

2. Nonlinear higher RPA by A. Smetana, F. Šimkovic, M. Krivorchenko, and J.T.
Two methods of QRPA approach under closure approx.

\[ M^{(0\nu)} \equiv \sum_{pp'nn'} \langle pp' | V(\bar{E}) | nn' \rangle \langle 0_f^+ | c_p^+, c_n, c_p^+ c_n | 0_i^+ \rangle \]

\[ \sum_{b_f: \text{pnQRPA}} |b_f \rangle \langle b_f| \sum_{b_i: \text{pnQRPA}} |b_i \rangle \langle b_i| \]

\[ M^{(0\nu)} \equiv \sum_{pp'nn'} \langle pp' | V(\bar{E}) | nn' \rangle \langle 0_f^+ | c_p^+, c_n, c_p^+ c_n | 0_i^+ \rangle \]

\[ \sum_{b_f: \text{likeQRPA}} |b_f \rangle \langle b_f| \sum_{b_i: \text{likeQRPA}} |b_i \rangle \langle b_i| \]
The overlap of QRPA states

The QRPA ground state $|0^+_{QRPA,i}\rangle$ is defined as the vacuum of the QRPA quasiboson:

$$O_b^i |0^+_{QRPA,i}\rangle = 0$$

$O_b^i$ : annihilation operator of QRPA state $b$

$$|0^+_{QRPA,i}\rangle = \prod_{K\pi} \frac{1}{N_{QRPA,i}^{K\pi}} \exp[v_i^{(K\pi)}] |0^+_{HFB,i}\rangle,$$

$$v_i^{(K\pi)} \cong \sum_{\mu\nu\mu'\nu'} \frac{1}{1 + \delta_{K0}} \left( \frac{Y_{i,K\pi}}{X_{i,K\pi}} \right)^\dagger a_{\mu}^{i\dagger} a_{\nu}^{i\dagger} a_{\mu'}^{i\dagger} a_{\nu'}^{i\dagger}$$

$$O_{b}^{i\dagger} = \sum_{\mu\nu\mu'\nu'} \left( X_{\mu\nu,b} a_{\mu}^{i\dagger} a_{\nu}^{i\dagger} - Y_{i,K\pi}^{i,K\pi} b a_{\nu}^{i} a_{\mu}^{i\dagger} \right),$$

$$a_{\mu}^{i} |0^+_{HFB,i}\rangle = 0.$$
Result for $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$

HFB gs is used instead of QRPA gs in the overlap calculations.

The value of my method

The product of the QRPA ground-state normalization factors = 1.84

$$M^{(0\nu)} = \sum_{K'=-\max K}^{\max K} \sum_{\pi} M^{(0\nu)}(K', \pi)$$
### Comparison ($^{150}\text{Nd} \rightarrow ^{150}\text{Sm}, g_A=1.25$)

<table>
<thead>
<tr>
<th></th>
<th>J. T.</th>
<th>Fang et al. (Tübingen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M^{(0\nu)}$</td>
<td>3.60</td>
<td>3.34</td>
</tr>
<tr>
<td>Method</td>
<td>Like-particle QRPA</td>
<td>PnQRPA</td>
</tr>
<tr>
<td>Residual interaction</td>
<td>Skyrme + volume pairing, no pn pairing</td>
<td>G matrix (CD Bonn) +pn pairing</td>
</tr>
<tr>
<td>Overlap calculation</td>
<td>1/normalization factors = 0.54</td>
<td>1/normalization factors = 1</td>
</tr>
</tbody>
</table>

The pn pairing interaction has an effect to reduce the NME.

D.-L. Fang et al., PRC **83**, 034320 (2011)
Two paths in QRPA approach under closure approx.

\[
\sum_{pp'nn'} \langle pp' | V(\bar{E}) | nn' \rangle \quad \sum_{d_fd_i:pnQRPA} \langle 0_f^+ | c_p^+, c_n | d_f \rangle \langle d_f | d_i \rangle \langle d_i | c_p^+ c_n | 0_i^+ \rangle
\]

\[
\sum_{pp'nn'} \langle pp' | V(\bar{E}) | nn' \rangle \quad \sum_{b_fb_i:likeQRPA} \langle 0_f^+ | c_p^+, c_p | b_f \rangle \langle b_f | b_i \rangle \langle b_i | c_n c_n' | 0_i^+ \rangle
\]

\[ \text{Pn-pairing int. is important for } \beta \text{ decay.} \]
\[ \text{Like-particle pairing int. is important for two-particle transfer.} \]

Overlap of QRPA states = \[ \prod_{f \ i} \text{(Normalization factors of the QRPA g.s.)} \]
The equivalence of the two different paths provides us with a constraint on the strengths of the effective interactions having different roles in the QRPA.

This principle → the strength of the $T=0$ pn-pairing int.

J.T. PRC 93, 024317 (2016)

Other interactions used:
Skyrme SkM*, like-particle pairing, and Coulomb interaction

<table>
<thead>
<tr>
<th>Pairing int. (MeV fm$^3$)</th>
<th>150Nd</th>
<th>150Sm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proton</td>
<td>Neutron</td>
</tr>
<tr>
<td>Like-ptcl.</td>
<td>-218.52</td>
<td>-176.36</td>
</tr>
<tr>
<td>$T=0$ (pn)</td>
<td>-197.44</td>
<td></td>
</tr>
<tr>
<td>$g_A$=$1.254$ (bare value)</td>
<td>$g_A$=$1.000$ (effective value)</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>$^{150}$Nd$\rightarrow^{150}$Sm</td>
<td>$^{2\nu}\beta\beta$ nuclear matrix element</td>
<td></td>
</tr>
<tr>
<td>My cal.</td>
<td>0.0816</td>
<td></td>
</tr>
<tr>
<td>Semiexp.</td>
<td>0.0368</td>
<td></td>
</tr>
<tr>
<td>My cal.</td>
<td>0.0849</td>
<td></td>
</tr>
<tr>
<td>Semiexp.</td>
<td>0.0579</td>
<td></td>
</tr>
</tbody>
</table>

- Usually the semiexp. $^{2\nu}\beta\beta$ nuclear matrix element is fitted by adjusting the strength of the pn pairing interaction in the QRPA approach.
- In my cal. that interaction strength is determined by an original theoretical method.
- Semiexp. value is obtained by the exp. half-life and phase-space factor including $g_A$. 
Second part: extension of RPA – under development

We aim at solving

the discrepancy problem of the nuclear matrix elements between the different methods

One of what we can do is

extension of RPA to higher-order particle-hole correlations

Our choice of method for the extension

Nonlinear higher RPA (nhRPA)

including the 2p-2h, … for expressing the excitations on top of the ground state
NhRPA equation \[ \text{arXiv:1701.08368} \]

Express excited state \( |\Psi_k\rangle \) as
\[
|\Psi_k\rangle = Q_k^\dagger |\Psi_0\rangle
\]

Ground state
\[
[H, Q_k^\dagger] |\Psi_0\rangle = E_{k0} Q_k^\dagger |\Psi_0\rangle
\]

Nonlinear and non-hermite eigeneq. in matrix-vector form (extension of the RPA eq.)

Solved by iteration

- Hamiltonian matrix elements \( \leftarrow |\Psi_0\rangle \)
- Eigenvector \( \rightarrow \) components of \( Q_k^\dagger \)
- Eigenvalue \( \rightarrow E_{k0} \)
- Solution vector \( \rightarrow \) components of \( |\Psi_0\rangle \)

D.J. Rowe, Rev. Mod. Phys. 40, 153 (1968)
**Lipkin model**

<table>
<thead>
<tr>
<th>Level index</th>
<th>Energy $\epsilon / 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\epsilon / 2$</td>
</tr>
<tr>
<td>0</td>
<td>$-\epsilon / 2$</td>
</tr>
</tbody>
</table>

$m = 1, \ldots, N$

**Energy**

\[ H = \epsilon J_z + \frac{V}{2} (J_+^2 + J_-^2) \]

**Level**

\[ J_z = \frac{1}{2} \sum_{m=1}^{N} (a_1^\dagger a_1 a_0^\dagger a_0 - a_0^\dagger a_0 a_1^\dagger a_1) \]

\[ J_+ = \sum_{m=1}^{N} a_1^\dagger a_0 \quad J_- = J_+^\dagger \]

Useful for test of theory, often used.

H.J. Lipkin et al., N.P. **62**, 188 (1965)

**Two subspace**

\[ \{ \left| \psi_0 \right\rangle, J_+^2 |\psi_0\rangle, \ldots, J_+^N |\psi_0\rangle \} \]

decoupled

\[ \{ J_+ |\psi_0\rangle, \ldots, J_+^{N-1} |\psi_0\rangle \} \]
Achievement 1

We found that nhRPA is equivalent to exact Schrödinger eq. by solving the equations for the first time.

\[
Q_{k}^{e+} = c_{k} + \sum_{l=1}^{N/2} \left( X_{2l}^{k} J_{+}^{2l} + Y_{2l}^{k} J_{-}^{2l} \right)
\]

This term has been overlooked by other groups years. Necessary for the subspace including the ground state.
Achievement 2

Comparison with shell model under truncation of dimension of matrix used in calculation

$d$: dimension of the matrix used in the calculation

$d$ of exact cal. = \(N/2 = 10\)
\[ Q_k^\dagger |\psi_0\rangle = \left[ \sum_{l=1}^{d} \left( X_{2l}^k J_{+}^{2l} + Y_{2l}^k J_{-}^{2l} \right) + c_k \right] \]

- The highest order of \( J_{+}^{2l} \) of excited state = 4d
- Corresponding order of shell model = 2d

Unperturbed ground state

0th component

P-h component

Eigeneq. with matrix of dimension \( d \)

Linear eq. with matrix of dimension \( d \)
Summary

1. Three originalities in calculation of $\beta\beta$ NME presented:
   
   i. Like-particle QRPA
   ii. Accurate overlap calculation
   iii. Theoretical determination of the strength of $T=0$ pairing interaction

   For $2\nu\beta\beta$ NME of $^{150}\text{Nd}$, Cal./semiexp = 1.47, ($g_A = 1.0$).

2. Extension of RPA presented: nonlinear higher RPA
   
   i. Equivalent to exact Schrödinger eq.
   ii. High performance under truncation of wavefunction space
   iii. Iteration necessary.