

Effective value of the axial-vector coupling constant in β and $\beta\beta$ decays

Jouni Suhonen

Department of Physics, University of Jyväskylä

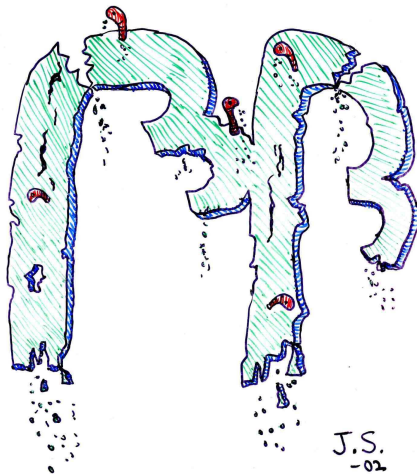
Matrix Elements for Double-beta-decay EXperiments (MEDEX'17)
Prague, Czech Republic, May 29 - June 2, 2017



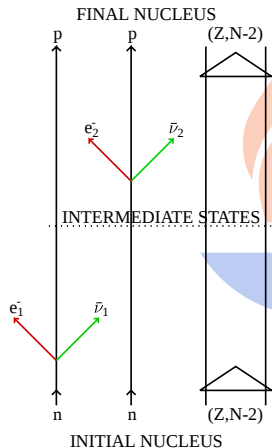
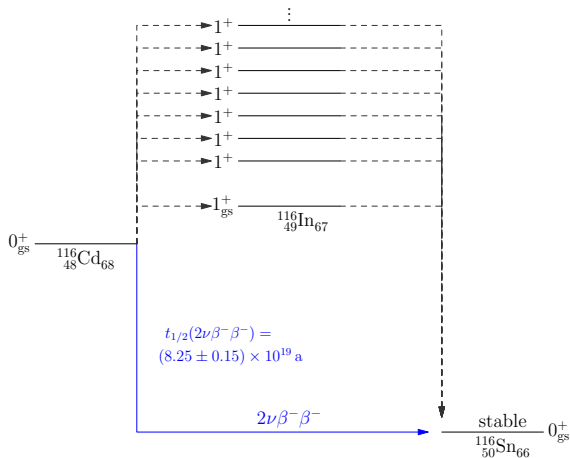
Contents:

- Incentive: Why Bother?
- GT and SD β Decays
- Unique Spin-Multipole Decays
- Nonunique forbidden β Decays (spectrum-shape method)
- ~~Isovector spin-multipole GRs~~

Motivation for the Work: Double Beta Decay

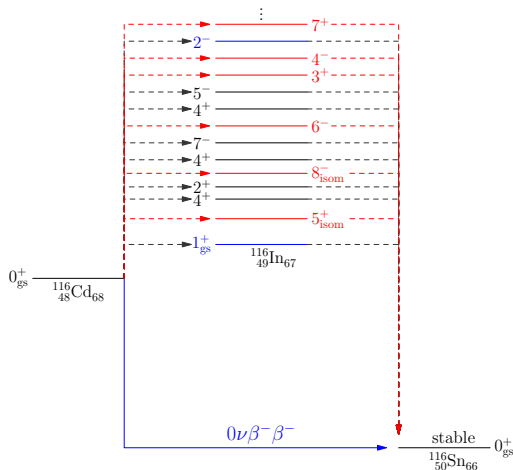


Two-Neutrino Double Beta Decay of ^{116}Cd

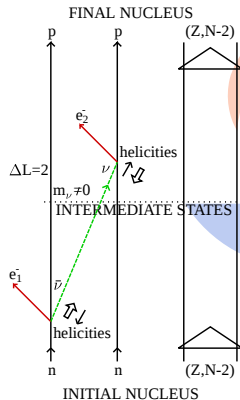


$$2\nu\beta\beta - \text{rate} \sim \left| M_{\text{GTGT}}^{(2\nu)} \right|^2 = (g_A)^4 \sum_{m,n} \frac{M_L(1_m^+) M_R(1_n^+)}{D_m}$$

Neutrinoless Double Beta Decay of ^{116}Cd



MASS MODE



$$0\nu\beta\beta - \text{rate} \sim \left| M_{\text{GTGT}}^{(0\nu)} \right|^2 = (g_{A,0\nu})^4 \left| \sum_{J^\pi} \langle 0_f^+ || \mathcal{O}_{\text{GTGT}}^{(0\nu)}(J^\pi) || 0_i^+ \rangle \right|^2$$

Definitions

Quenching:

$$q = g_A / g_A^{\text{free}}$$

Free value of g_A :

$$g_A^{\text{free}} = 1.27$$

Effective value of g_A :

$$g_A^{\text{eff}} = q g_A^{\text{free}}$$

Gamow-Teller β decays

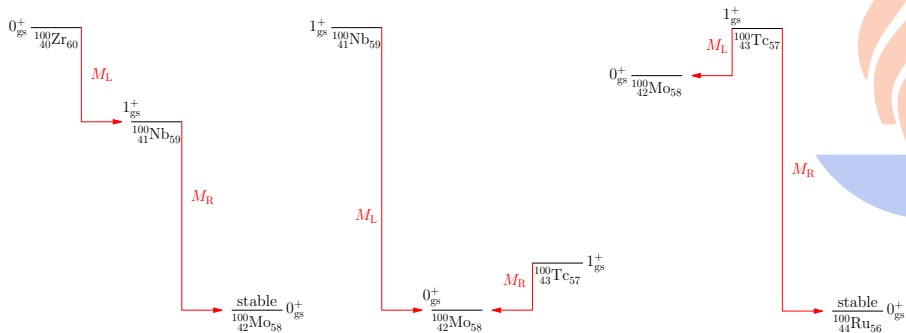
There are data on:

Gamow-Teller β TRANSITIONS

Theoretical approaches:

ISM (Interacting Shell Model)
pnQRPA (proton-neutron QRPA)

Typical Gamow-Teller β transitions



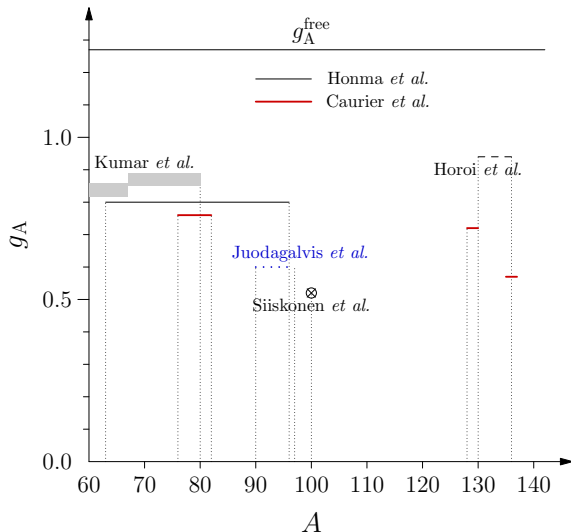
Results from:

Quenching of g_A in the ISM calculations

Results from the ISM

Mass range	g_A^{eff}	Reference
Full $0p$ shell	$1.03^{+0.03}_{-0.02}$	W. T. Chou <i>et al.</i> 1993
$0p - \text{low } 1s0d$ shell	$1.12^{+0.05}_{-0.04}$	D. H. Wilkinson <i>et al.</i> 1974
Full $1s0d$ shell	$0.96^{+0.03}_{-0.02}$	B. H. Wildenthal <i>et al.</i> 1983
	1.0	T. Siiskonen <i>et al.</i> 2001
$A = 41 - 50$ ($1p0f$ shell)	$0.937^{+0.019}_{-0.018}$	G. Martínez-Pinedo <i>et al.</i> 1996
$1p0f$ shell	0.98	T. Siiskonen <i>et al.</i> 2001
^{56}Ni	0.71	T. Siiskonen <i>et al.</i> 2001
$A = 52 - 67$ ($1p0f$ shell)	$0.838^{+0.021}_{-0.020}$	V. Kumar <i>et al.</i> 2016
$A = 67 - 80$ ($0f_{5/2}1p0g_{9/2}$ shell)	0.869 ± 0.019	V. Kumar <i>et al.</i> 2016
$A = 63 - 96$ ($1p0f0g1d2s$ shell)	0.8	M. Honma <i>et al.</i> 2006
$A = 76 - 82$ ($1p0f0g_{9/2}$ shell)	0.76	E. Caurier <i>et al.</i> 2012
$A = 90 - 97$ ($1p0f0g1d2s$ shell)	0.60	A. Juodagalvis <i>et al.</i> 2005
^{100}Sn	0.52	T. Siiskonen <i>et al.</i> 2001
$A = 128 - 130$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.72	E. Caurier <i>et al.</i> 2012
$A = 130 - 136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.94	M. Horoi <i>et al.</i> 2016
$A = 136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.57	E. Caurier <i>et al.</i> 2012

Results from the ISM (illustration)



- Kumar *et al.*: J. Phys. G 43 (2016) 105104
- Honma *et al.*: J. Phys. Conf. Ser. 49 (2006) 45
- Caurier *et al.*: Phys. Lett. B 711 (2012) 62
- Horoi *et al.*: Phys. Rev. C 93 (2016) 024308
- Juodagalvis *et al.*: Phys. Rev. C 72 (2005) 024306
- Siiskonen *et al.*: Phys. Rev. C 63 (2001) 055501

Proton-neutron Quasiparticle Random-Phase Approximation (pnQRPA)

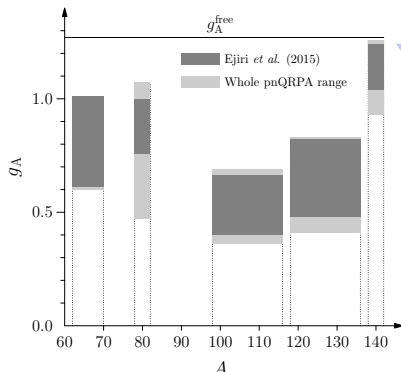
Results from:

Quenching of g_A
in the pnQRPA calculations

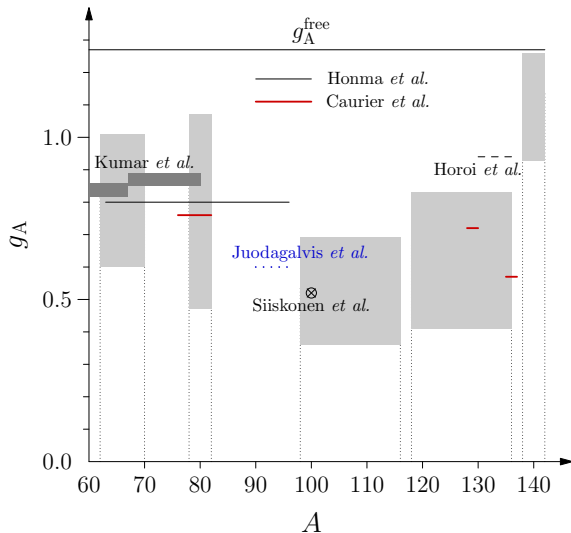
Results from the pnQRPA

A	pn Conf.	$\bar{g}_A^{\text{eff}}$				$\bar{g}_{pp}$ [1]
		[2]	[1] (tripl.)	[1] (mult.)	[3]	
62 – 70	$1p_{3/2} - 1p_{1/2}$	0.81 ± 0.20	0.80 ± 0.20	0.84 ± 0.15	-	0.71 ± 0.34
78 – 82	$0g_{9/2} - 0g_{7/2}$	0.88 ± 0.12	0.77 ± 0.30	(0.87 ± 0.74)	-	0.53 ± 0.33
98 – 116	$0g_{9/2} - 0g_{7/2}$	0.53 ± 0.13	0.54 ± 0.15	0.53 ± 0.14	0.52 ± 0.16	0.74 ± 0.17
118 – 136	$1d_{5/2} - 1d_{5/2}$	0.65 ± 0.17	0.65 ± 0.16	0.59 ± 0.18	0.67 ± 0.16	0.56 ± 0.24
62 – 136		0.72 ± 0.16	0.69 ± 0.12	0.71 ± 0.17	0.60 ± 0.11	0.63 ± 0.11
138 – 142	$1d_{5/2} - 1d_{3/2}$	1.14 ± 0.10	1.13 ± 0.13	1.07 ± 0.14	-	0.59 ± 0.11

- [1] F. Deppisch, J. Suhonen,
Phys. Rev. C 94 (2016) 055501
- [2] H. Ejiri, J. Suhonen, J. Phys.
G 42 (2015) 055201
- [3] P. Pirinen, J. Suhonen, Phys.
Rev. C 91 (2015) 054309



Results from the ISM on top of the pnQRPA ranges



- Kumar *et al.*: J. Phys. G 43 (2016) 105104
- Honma *et al.*: J. Phys. Conf. Ser. 49 (2006) 45
- Caurier *et al.*: Phys. Lett. B 711 (2012) 62
- Horoi *et al.*: Phys. Rev. C 93 (2016) 024308
- Juodagalvis *et al.*: Phys. Rev. C 72 (2005) 024306
- Siiskonen *et al.*: Phys. Rev. C 63 (2001) 055501

Calculations for the β decays and $\beta\beta$ decays

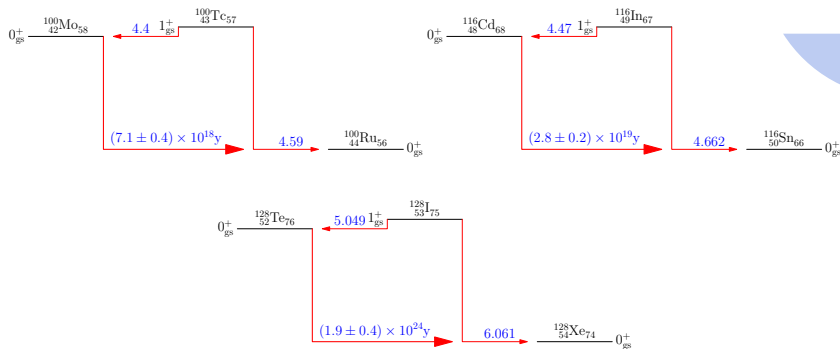
Results from:

Quenching of g_A
in the pnQRPA-based,
ISM-based and
IBM-based calculations
of β decays and $\beta\beta$ decays

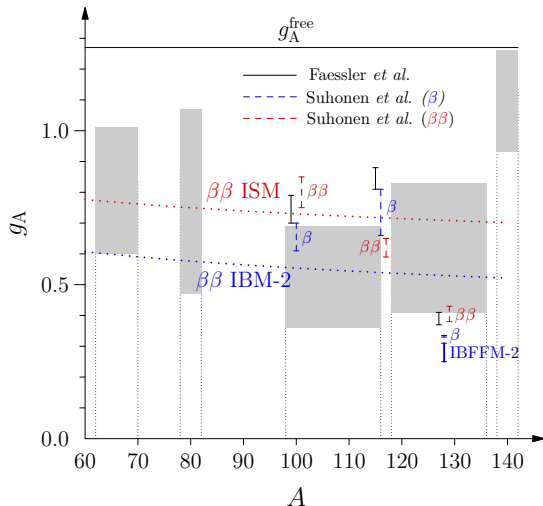
Results from the pnQRPA, IBM-2, and IBFFM-2

A	pnQRPA			IBFFM-2 [1]		IBM-2 [2]
	$g_A(\beta + \beta\beta)$ [3]	$g_A(\beta)$ [4]	$g_A(\beta\beta)$ [4]	$g_A(\beta)$	$g_A(\beta\beta)$	$g_A(\beta\beta)$
100	0.70 – 0.79	0.61 – 0.70	0.75 – 0.85	-	-	0.46(1) [SSD]
116	0.81 – 0.88	0.66 – 0.81	0.59 – 0.65	-	-	0.41(1) [SSD]
128	0.37 – 0.41	0.330 – 0.335	0.38 – 0.43	0.25 – 0.31	0.293	0.55(3) [CA]

[1] N. Yoshida, F. Iachello, Prog. Theor. Exp. Phys. 2013 (2013) 043D01 ; [2] J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 87 (2013) 014315 ; [3] A. Faessler *et al.*, arXiv 0711.3996v1 [Nucl-th] ; [4] J. Suhonen, O. Civitarese, Nucl. Phys. A 924 (2014) 1



Results from the $\beta+\beta\beta$ calculations against the pnQRPA ranges from Gamow-Teller β decays



- **Faessler *et al.***: A. Faessler, G. L. Fogli, E. Lisi, V. Rodin, A. M. Rotunno, F. Šimkovic, arXiv 0711.3996v1 [Nucl-th]
- **Suhonen *et al.***: J. Suhonen, O. Civitarese, Nucl. Phys. A 924 (2014) 1
- **$\beta\beta$ ISM and IBM-2**: J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 87 (2013) 014315

Results from:

Quenching of g_A
as derived from
spin-multipole NMEs
of forbidden unique β decays

Spin-multipole (SM) nuclear matrix elements

General half-life formula for the **allowed** and **unique-forbidden** beta decays

$$t_{1/2}^K(0_{\text{gs}}^+ \leftrightarrow J^\pi) = \frac{\text{Constant}}{\frac{g_A^2}{2J_i+1} (M^K(\text{SM}J^\pi))^2 f_K},$$

where

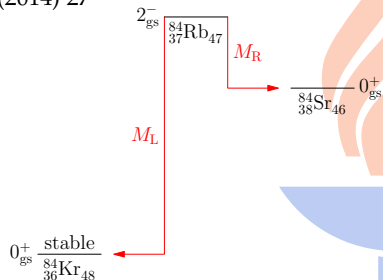
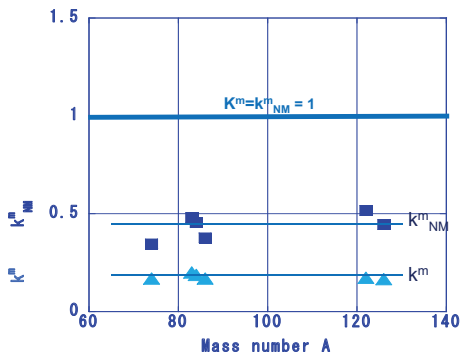
- f_K is the phase-space factor for the K^{th} **forbidden** (allowed $\equiv 0^{\text{th}}$ forbidden) **unique** β -decay transition,
- g_A is the axial-vector coupling constant,
- $J_i = J$ or $J_i = 0$ ($J = K + 1$) is the angular momentum of the decaying state, and
- $M^K(\text{SM}J^\pi)$ is the spin-multipole NME for the K^{th} **forbidden unique** transition.

The unique decays are classified as:

K	0 (allowed)	1	2	3	4	5	6	7
J^π	1^+	2^-	3^+	4^-	5^+	6^-	7^+	8^-

Global study for the first-forbidden ($K = 1$) spin-dipole $2^-_{\text{gs}} \rightarrow 0^+_{\text{gs}}$ decays

H. Ejiri, N. Soukouti and J. Suhonen, Spin-dipole nuclear matrix elements for double beta decays and astro-neutrinos, Phys. Lett. B 729 (2014) 27



$$\bar{M}(\text{SD}2^-) = \sqrt{M_L M_R}$$

$$\langle k \rangle = \left\langle \frac{\bar{M}_{\text{exp}}(\text{SD}2^-)}{M_{\text{qp}}(\text{SD}2^-)} \right\rangle \approx 0.18$$

$$\langle k_{\text{NM}} \rangle = \left\langle \frac{\bar{M}_{\text{exp}}(\text{SD}2^-)}{M_{\text{pnQRPA}}(\text{SD}2^-)} \right\rangle \approx 0.45$$

Decays through higher spin-multipole ($K \geq 2$) operators

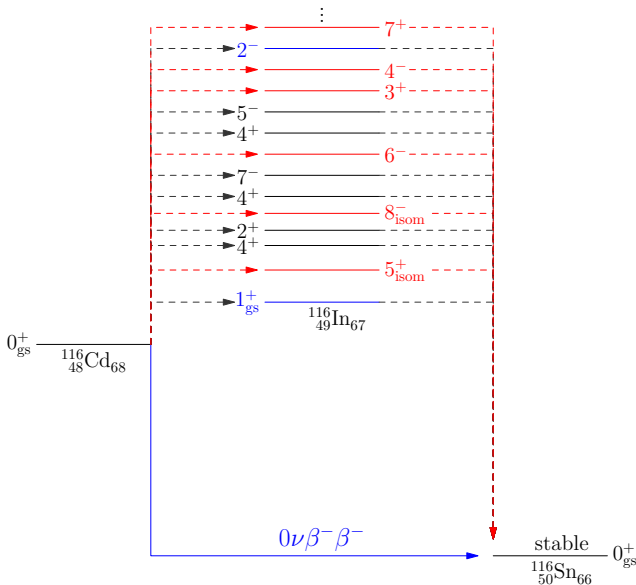
Question:

WHAT CAN WE LEARN
FROM THE UNIQUE HIGHER-FORBIDDEN
 β DECAYS?

Answer:

A LOT!

INCENTIVE: $0\nu\beta\beta$ decay through the higher spin-multipole states



Decays through higher spin-multipole ($K \geq 2$) operators

Task:

STUDY 148 UNIQUE HIGHER-FORBIDDEN
 β DECAYS IN ISOTOPIC CHAINS

Problem:

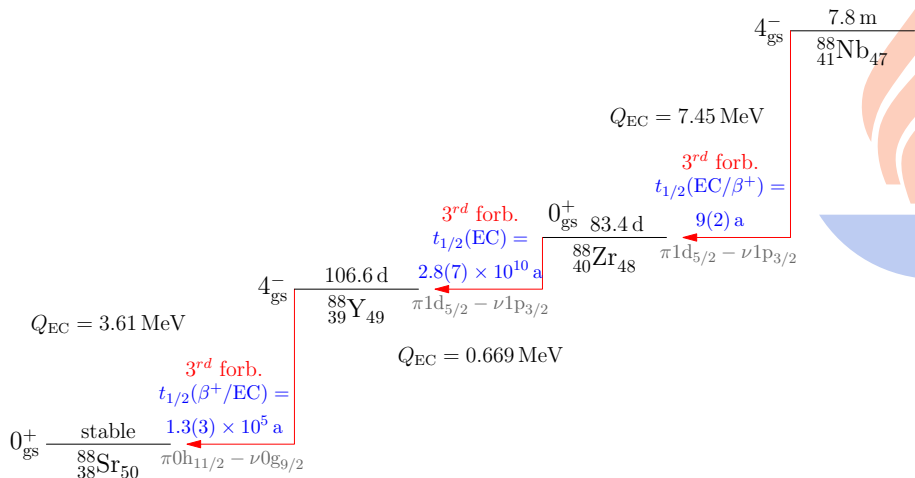
NO EXP. DATA AVAILABLE

Study:

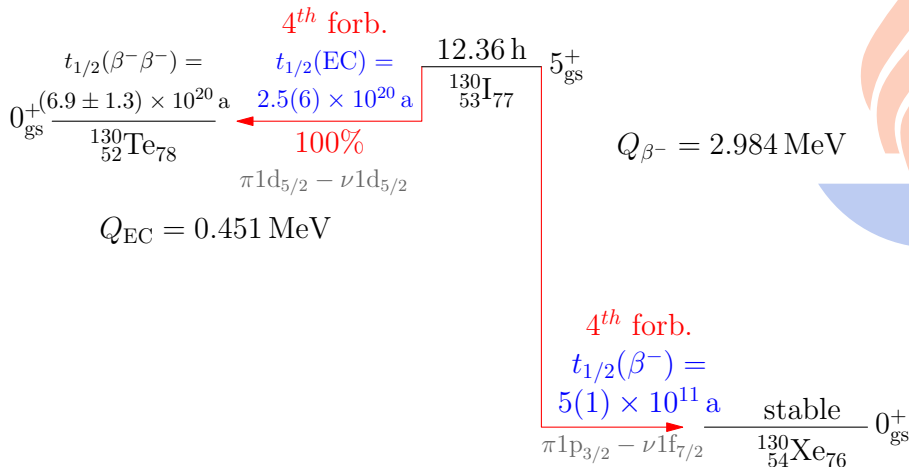
$$k = \frac{M_{\text{pnQRPA}}^K(\text{SMJ}^\pi)}{M_{\text{qp}}^K(\text{SMJ}^\pi)} = ?$$

Dependence on K and mass number A ?

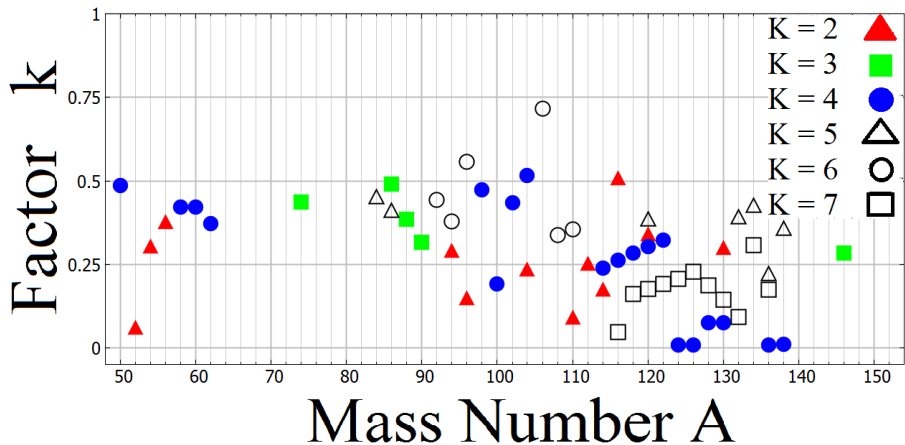
Example: Decays in the $A = 88$ chain



Example: Decays in the $A = 130$ chain (including a $\beta\beta$ decay)

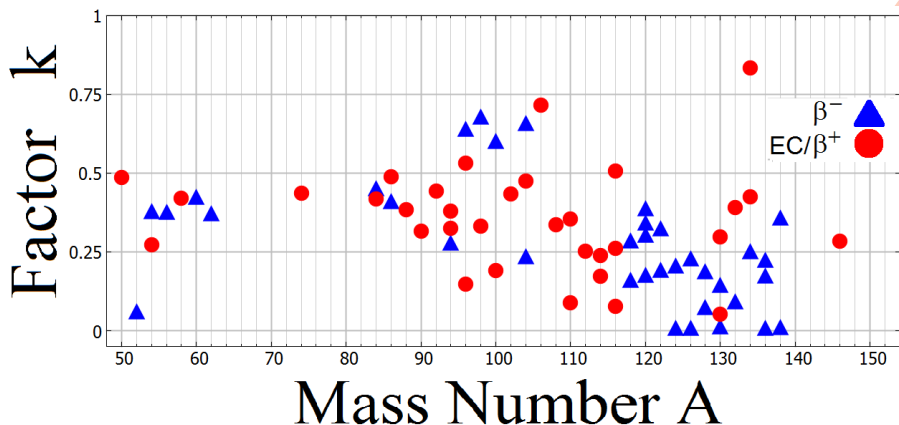


Ratio k for 74 β decays involving non-magic nuclei



k extracted using the **geometric mean** of the full set of K^{th} ($K = 2 - 7$) forbidden β -decay transitions in an isobaric chain (J. Kostensalo, J. Suhonen, Phys. Rev. C 95 (2017) 014322)

Separation to β^- and β^+/EC decays



Results for the Ratio $k = M_{\text{pnQRPA}}^K(\text{SMJ}^\pi) / M_{\text{qp}}^K(\text{SMJ}^\pi)$

A	GT [1]	K = 1 [2]	K = 2	K = 3	K = 4	K = 5	K = 6	K = 7	Avg.
50 – 88	0.35	0.40	0.25	0.46	0.43	0.43	-	-	0.39
90 – 122	0.52	0.40	0.25	0.35	0.34	0.38	0.41	0.13	0.31
122 – 146	0.40	0.40	0.30	0.28	0.07	0.35	-	0.19	0.24
Average	0.42	0.40	0.27	0.36	0.28	0.39	0.41	0.16	0.31

[1] H. Ejiri, J. Suhonen, J. Phys. G: Nucl. Part. Phys. 42 (2015) 055201

[2] H. Ejiri, N. Soukouti, J. Suhonen, Phys. Lett. B 729 (2014) 27

Conclusion: k is roughly independent of $K \Rightarrow$ Low-energy quenching of g_A derivable from the hatched regions of the Gamow-Teller studies in the pnQRPA framework:

Mass range	A = 76 – 82	A = 100 – 116	A = 122 – 136
$g_{A,0\nu}^{\text{eff}}$	0.7 – 0.9	0.5	0.5 – 0.7

Assumption: Also the forbidden non-unique virtual transitions behave like the forbidden unique virtual transitions.

Spectrum-shape method (SSM)

Results from:

Quenching of g_A
as derived from
electron spectra of
forbidden non-unique β decays

Spectrum shape of forbidden non-unique β decays

Half-life:

$$t_{1/2} = \kappa / \tilde{C}.$$

Dimensionless integrated shape function:

$$\tilde{C} = \int_1^{w_0} C(w_e) p w_e (w_0 - w_e)^2 F_0(Z_f, w_e) dw_e.$$

Shape factor:

$$C(w_e) = \sum_{k_e, k_\nu, K} \lambda_{k_e} \left[M_K(k_e, k_\nu)^2 + m_K(k_e, k_\nu)^2 - \frac{2\gamma_{k_e}}{k_e w_e} M_K(k_e, k_\nu) m_K(k_e, k_\nu) \right],$$

where

$$\lambda_{k_e} = \frac{F_{k_e-1}(Z, w_e)}{F_0(Z, w_e)}; \quad \gamma_{k_e} = \sqrt{k_e^2 - (\alpha Z_f)^2},$$

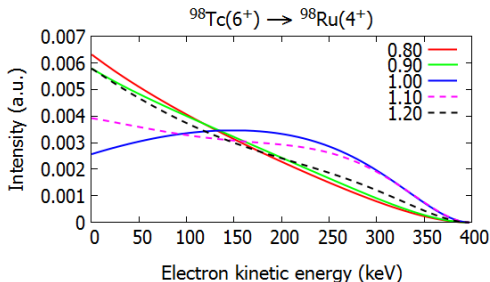
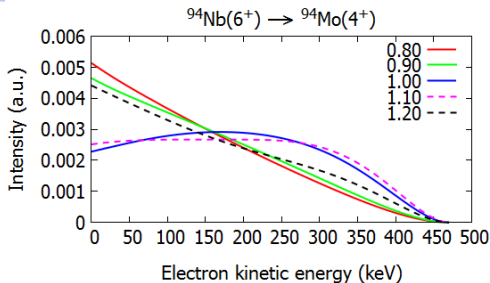
$F_{k-1}(Z, w_e)$ being the generalized Fermi function.

Decomposition of the shape factor:

$$C(w_e) = g_V^2 C_V(w_e) + g_A^2 C_A(w_e) + g_V g_A C_{VA}(w_e).$$

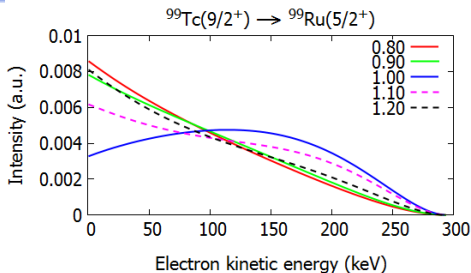
ISM-computed β spectra for different values of g_A

Normalized
ISM-computed
electron spectra for
the $2nd$ -forbidden
nonunique β^-
decays of ^{94}Nb and
 ^{98}Tc ($g_V = 1.0$).

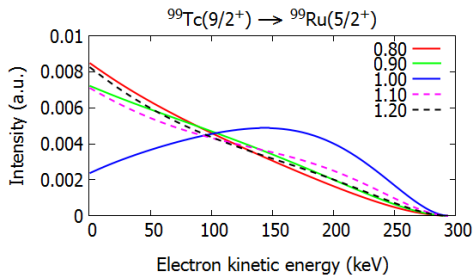


Example: ISM- and MQPM-computed electron spectra

Normalized ISM-
and
MQPM-computed
electron spectra for
the 2nd-forbidden
nonunique β^- decay
of ^{99}Tc ($g_V = 1.0$)
using different
values of g_A .



(ISM)



(MQPM)

Example: Decay of ^{113}Cd – Comparison with data

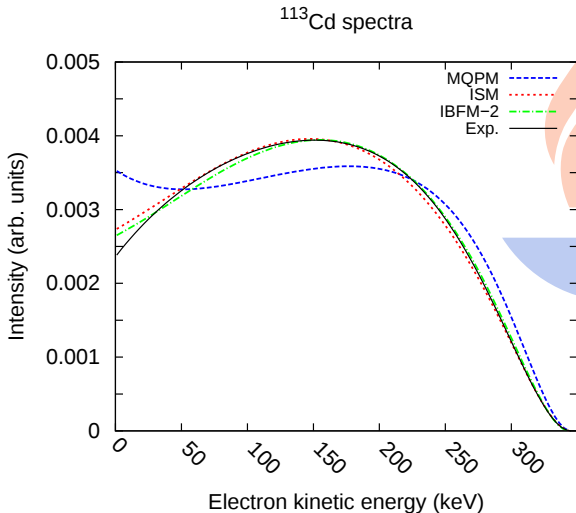
Normalized electron spectra
for the 4th-forbidden
nonunique β^- decay
 $^{113}\text{Cd}(1/2^+) \rightarrow ^{113}\text{In}(9/2^+)$
($g_V = 1.0$).

Experimental data from:

P. Belli *et al.*, Phys. Rev. C 76
(2007) 064603

All three nuclear models
give:

$$g_A \approx 0.92!$$



Summary of the exploratory work on β spectra

Transition	$J_i^{\pi_i}$ (gs)	$J_f^{\pi_f}$ (n_f)	Branching	K	Sensitivity	Nucl. model
$^{36}\text{Cl} \rightarrow ^{36}\text{Ar}$	2 ⁺	0 ⁺ (gs)	98%	2	None	ISM
$^{48}\text{Ca} \rightarrow ^{48}\text{Sc}$	0 ⁺	4 ⁺ (2)	$\sim 0\%$	4	None	ISM
$^{48}\text{Ca} \rightarrow ^{48}\text{Sc}$	0 ⁺	6 ⁺ (gs)	$\sim 0\%$	6	None	ISM
$^{50}\text{V} \rightarrow ^{50}\text{Cr}$	6 ⁺	2 ⁺ (1)	$\sim 0\%$	4	Weak	ISM
$^{60}\text{Fe} \rightarrow ^{60}\text{Co}$	0 ⁺	2 ⁺ (1)	100%	2	None	ISM
$^{85}\text{Br} \rightarrow ^{85}\text{Kr}$	3/2 ⁻	9/2 ⁺ (gs)	$\sim 0\%$	3	Moderate	MQPM
$^{87}\text{Rb} \rightarrow ^{87}\text{Sr}$	3/2 ⁻	9/2 ⁺ (gs)	100%	3	Moderate	MQPM, ISM
$^{93}\text{Zr} \rightarrow ^{93}\text{Nb}$	5/2 ⁺	9/2 ⁺ (gs)	$5 \leq$	2	Weak	MQPM
$^{94}\text{Nb} \rightarrow ^{94}\text{Mo}$	6 ⁺	4 ⁺ (2)	100%	2	Strong	NSM
$^{96}\text{Zr} \rightarrow ^{96}\text{Nb}$	0 ⁺	4 ⁺ (2)	$\sim 0\%$	4	None	ISM
$^{96}\text{Zr} \rightarrow ^{96}\text{Nb}$	0 ⁺	6 ⁺ (gs)	$\sim 0\%$	6	Strong	ISM
$^{97}\text{Zr} \rightarrow ^{97}\text{Nb}$	1/2 ⁺	9/2 ⁺ (gs)	$\sim 0\%$	4	Strong	MQPM
$^{98}\text{Tc} \rightarrow ^{98}\text{Ru}$	6 ⁺	4 ⁺ (3)	100%	2	Strong	ISM
$^{99}\text{Tc} \rightarrow ^{99}\text{Ru}$	9/2 ⁺	5/2 ⁺ (gs)	100%	2	Strong	MQPM, ISM
$^{101}\text{Mo} \rightarrow ^{101}\text{Tc}$	1/2 ⁺	9/2 ⁺ (gs)	$\sim 0\%$	4	Strong	MQPM
$^{113}\text{Cd} \rightarrow ^{113}\text{In}$	1/2 ⁺	9/2 ⁺ (gs)	100%	4	Strong	MQPM, ISM, IBFM-2
$^{115}\text{Cd} \rightarrow ^{115}\text{In}$	1/2 ⁺	9/2 ⁺ (gs)	$\sim 0\%$	4	Strong	MQPM
$^{115}\text{In} \rightarrow ^{115}\text{Sn}$	9/2 ⁺	1/2 ⁺ (gs)	100%	4	Strong	MQPM, ISM, IBFM-2

Summary on β spectra continues

Transition	$J_i^{\pi_i}$ (gs)	$J_f^{\pi_f}$ (n_f)	Branching	K	Sensitivity	Nucl. model
$^{117}\text{Cd} \rightarrow ^{117}\text{In}$	$1/2^+$	$9/2^+$ (gs)	$\sim 0\%$	4	Strong	MQPM
$^{119}\text{In} \rightarrow ^{119}\text{Sn}$	$9/2^+$	$1/2^+$ (gs)	$\sim 0\%$	4	Strong	MQPM
$^{123}\text{Sn} \rightarrow ^{123}\text{Sb}$	$11/2^-$	$1/2^+$ (4)	$\sim 0\%$	5	Weak	MQPM
$^{126}\text{Sn} \rightarrow ^{126}\text{Sb}$	0^+	2^+ (5)	100%	2	None	ISM
$^{135}\text{Cs} \rightarrow ^{135}\text{Ba}$	$7/2^+$	$3/2^+$ (gs)	100%	2	None	MQPM
$^{137}\text{Cs} \rightarrow ^{137}\text{Ba}$	$7/2^+$	$3/2^+$ (gs)	5.4%	2	None	MQPM, ISM
$^{125}\text{Sb} \rightarrow ^{125}\text{Te}$	$7/2^+$	$9/2^-$ (3)	7.2%	1	None	MQPM
$^{141}\text{Ce} \rightarrow ^{141}\text{Pr}$	$7/2^-$	$5/2^+$ (gs)	31%	1	Weak	MQPM
$^{159}\text{Gd} \rightarrow ^{159}\text{Tb}$	$3/2^-$	$5/2^+$ (1)	26%	1	None	MQPM
$^{161}\text{Tb} \rightarrow ^{161}\text{Dy}$	$3/2^+$	$5/2^-$ (1)	$\sim 0\%$	1	None	MQPM
$^{169}\text{Er} \rightarrow ^{169}\text{Tm}$	$1/2^-$	$3/2^+$ (1)	45%	1	None	MQPM

Conclusions and Outlook

Conclusions:

- The long chain of ISM calculations and the recent pnQRPA and IBM-2 calculations of Gamow-Teller β decays and $2\nu\beta\beta$ decays are (surprisingly!) **consistent with each other** and clearly point to a **A-dependent quenched g_A**
- Previous studies on GT 1^+ and SD 2^- β decays shed light on the **suppression chain**: **quasiparticle NME** $\rightarrow$ **pnQRPA NME** $\rightarrow$ **experimental NME**
- From studies of **unique high-forbidden** β decays ($K \geq 2$) the **suppression chain**: **quasiparticle NME** $\rightarrow$ **pnQRPA NME** can be extracted $\rightarrow$ **expected half-lives** for the 148 studied **unique high-forbidden** β decays $\rightarrow$ one can speculate about modifications in the pnQRPA computed **$0\nu\beta\beta$ -decay half-lives**
- The **spectrum-shape method (SSM)** for forbidden non-unique β decays is a **robust tool** (largely independent of the nuclear model, the assumed Hamiltonian and mean field) to search for the **effective value of g_A**

Outlook:

- Urge **measurements of the β spectra** for the (5) interesting decays amenable to the SSM
- Find ways to use the present studies in a more **reliable prediction** of the **pnQRPA-based $0\nu\beta\beta$ NMEs**