New calculations
for Double Beta Decay

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Outline

- Introduction – DBD a nuclear rare decay with broad potential to test BSM physics
- Calculations of products $G^{(2,0)\nu} \times M^{(2,0)\nu}$
- Constraints for Lorentz violation parameters from the DBD study
- Conclusions

Introduction

Double beta decay (DBD) is the nuclear process with the longest lifetime measured so far, which presents a great interest particularly for testing LNV and understanding the neutrino properties.

According to the number and type of the released leptons we may have the following DBD modes:

$2\nu\beta^-\beta^-$; $0\nu\beta^-\beta^-$; $2\nu\beta^+\beta^+$; $0\nu\beta^+\beta^+$; $2\nu EC\beta^+$; $0\nu EC\beta^+$; $2\nu ECEC$; $0\nu ECEC$

•Study of neutrinoless DBD has a broad potential to test BSM physics

•Lepton number conservation $0\nu\beta\beta$: $(A, Z) \rightarrow (A, Z + 2) + 2\, e^-$

•Limits on different BSM parameters associated with different possible scenarios which may contribute to $0\nu\beta\beta$

- on the effective Majorana neutrino mass: $<m_{\beta\beta}>^2 = |\sum_i |U_{ei}|^2 m_i|^2$; $(U_{\alpha i}) \equiv$ PMNS matrix

in corroboration with ν oscillation data → hints on the ν mass hierarchy;

ν oscillation data (analysis of the $<m_{\beta\beta}> = f(m_0)$, with $m_0 = \min[m_1, m_2, m_3]$)

=> the existence of a lower limit ~0.014 eV for the Majorana m_ν for IH, while for NH

$<m_{\beta\beta}>$ could vanish

- limits on the RH currents couplings: in the hypothesis of existence of RH components of the weak interaction currents their strength can be characterized by phenomenological coupling constants η and λ (η describes the coupling between the RH lepton current and LH quark current, while λ describes the coupling when both currents are RH).

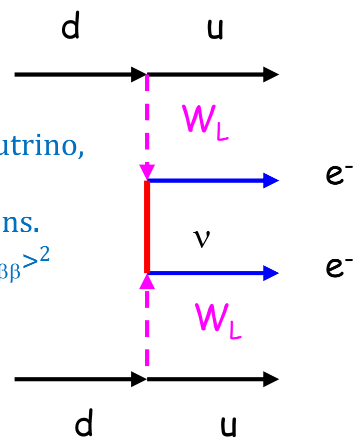
The observation of the single electron spectra could, in principle, allow to distinguish this mechanism from the light LH Majorana ν exchange mode.

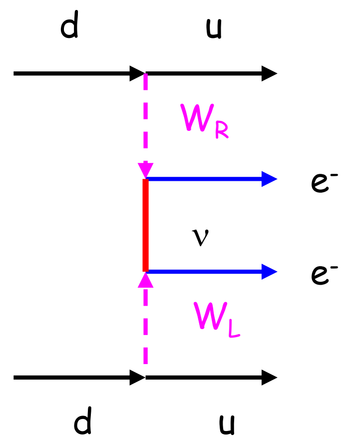
- limits on parameters related to other possible mechanisms mediating $0\nu\beta\beta$: Majoron, SUSY, etc.

0νββ-broad potential to search for beyond SM physics: any ΔL=2 process can contribute to 0νββ

Diagrams that can contribute to the 0νββ decay amplitude

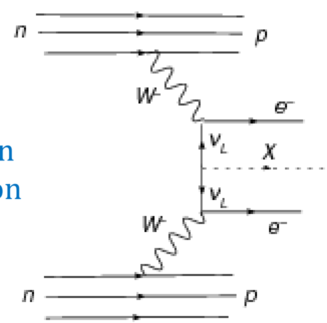
Light Majorana neutrino,
only SM LH
weak interactions.
Decay rate ~$<m_{\beta\beta}>^2$





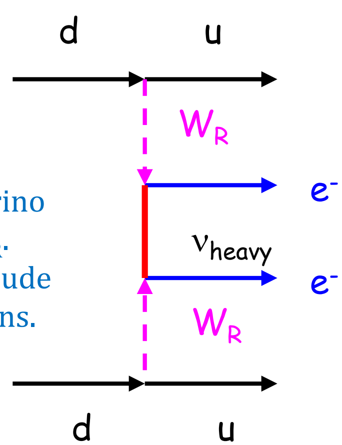
Light Majorana
neutrinos. Model extended
to include RH W_R.
Decay rate ~λ^2, η^2, $\lambda\eta$

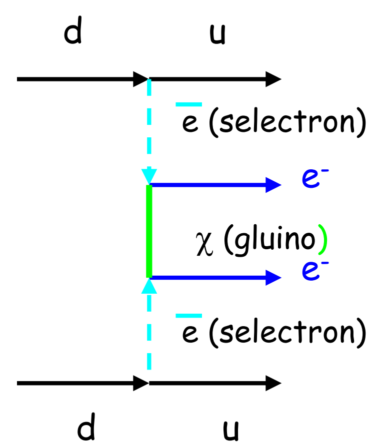
Majoron
emission



(a)

Heavy Majorana neutrino
interacting with W_R.
Model extended to include
RH current interactions.
Decay rate ~ η^2_N





SUSY with R-parity
violation. Many
new particles invoked.
Decay rate ~ η^2_g

- Test of the Lorentz violation (LV) in the neutrino sector

$2\nu\beta\beta$: to investigate modifications of the electron sum energy spectrum produced by isotropic LV and distortions of the one electron spectra for experiments with tracking systems that allow the determination of the direction of the two emitted electrons.

$0\nu\beta\beta$: LV Majorana couplings modify the ν propagator, introducing novel effects in $0\nu\beta\beta$; *one of such effects is that $0\nu\beta\beta$ might occur even in the absence of Majorana neutrino exchange mechanism, but only through the Lorentz violating mechanism*

For $2\nu\beta\beta$ one can constrain parameters related to the deformation of the electron spectra, while for $0\nu\beta\beta$ one can constrain parameters related to the possible occurrence of this decay mode in the event when $\langle m_{\beta\beta} \rangle$ vanishes

Uncertainties in the theoretical calculations for DBD

NME:

i) method of calculation (pnQRPA, ShM, IBA2, PHFB, GCM with EDF, etc.) differing by the choice and building of the model spaces and type of correlations taken into account

ii) nuclear approximations involved in calculations: FNS, SRC(parametrization in different ways of the correlation function), nuclear currents (higher order terms-tensor contribution, RH components), closure/non-closure approximation, etc.

iii) nuclear parameters: - $R_A = r_0 A^{1/3}$ (r_0 = (1.1 or 1.2) fm $(1.2/1.1)^2 \sim 1.19 \rightarrow \sim 20\%$
- $\langle E_N \rangle$ (average energy for the int. states in the odd-odd nucleus)~(2-5)%
- Λ_A , Λ_B cut-off parameters used for inclusion of FNS ~ 7%
- g_A (1.0 = quark value; 1.27=free nucleon value; quenched value ~(0.4-0.9)

PSF:

i) method of calculation of the electron w.f. (Primakoff&Rosen- non-relativistic approx., Tomoda, Doi-Kotani, Suhonen&Civitarese-relativistic, but approx. w.f.; Iachello, Stoica- exact Dirac functions
ii) accuracy of resolution of the Dirac equations and integration of the PSF

The NME and PSF values provided by different groups have produced some confusion/difficulty in interpreting the experimental results due to the different units in which they were provided and/or due to the use of different nuclear approximations and input parameters.

$2\nu\beta^-\beta^-$

$$G_{2\nu}^{\beta\beta}(0^+ \to 0^+) = \frac{2\tilde{A}^2}{3\ln 2 g_A^4 (m_e c^2)^2} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} d\epsilon_1 \int_{m_e c^2}^{Q^{\beta\beta}+2m_e c^2-\epsilon_1} d\epsilon_2 \int_0^{Q^{\beta\beta}+2mc_e^2-\epsilon_1-\epsilon_2} d\omega_1$$

$$\times f_{11}^{(0)} w_{2\nu} (\langle K_N \rangle^2 + \langle L_N \rangle^2 + \langle K_N \rangle \langle L_N \rangle)$$

$$\langle K_N \rangle = \frac{1}{\epsilon_1 + \omega_1 + \langle E_N \rangle - E_I} + \frac{1}{\epsilon_2 + \omega_2 + \langle E_N \rangle - E_I}$$

$$\langle L_N \rangle = \frac{1}{\epsilon_1 + \omega_2 + \langle E_N \rangle - E_I} + \frac{1}{\epsilon_2 + \omega_1 + \langle E_N \rangle - E_I}$$

$$w_{2\nu} = \frac{g_A^4 (G \cos\theta_C)^4}{64\pi^7 \hbar} \omega_1^2 \omega_2^2 (p_1 c)(p_2 c) \epsilon_1 \epsilon_2,$$

$0\nu\beta^{-}\beta^{-}$

$$G_{0\nu}^{\beta\beta}(0^+ \to 0^+) = \frac{2}{4 g_A^4 R_A^2 \ln 2} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} f_{11}^{(0)} w_{0\nu} d\epsilon_1 \qquad w_{0\nu} = \frac{g_A^4 (G\cos\theta_C)^4}{16\pi^5}(m_e c^2)^2 (\hbar c^2)(p_1 c)(p_2 c)\epsilon_1 \epsilon_2$$

$$M_\alpha^{0\nu} = \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTD\,(j_p j_{p'}, j_n j_{n'}; J_\pi)\,\langle j_p j_{p'}; J_\pi \| \tau_{-1}\tau_{-2} O_{12}^{\alpha} \| j_n j_{n'}; S_\alpha J_\pi \rangle$$

$$O_{12}^{GT} = \sigma_1 \cdot \sigma_2 H(r) \qquad O_{12}^{F} = H(r) \qquad O_{12}^{T} = \sqrt{\frac{2}{3}}\,[\sigma_1 \times \sigma_2]^2 \cdot \frac{r}{R} H(r) C^{(2)}(\hat{r})$$

$$H_\alpha(r) = \frac{2R}{\pi}\int_0^\infty j_i(qr)\frac{h_\alpha(q)}{\omega}\frac{1}{\omega + \langle E \rangle} q^2 dq \equiv \int_0^\infty j_i(qr) V_\alpha(q) q^2 dq$$

$$h_{GT}(q^2) = \frac{G_A^2(q^2)}{g_A^2}\left[1 - \frac{2}{3}\frac{q^2}{q^2+m_\pi^2} + \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right] + \frac{2}{3}\frac{G_M^2(q^2)}{g_A^2}\frac{q^2}{4m_p^2} \qquad h_F = G_V^2(q^2)$$

$$h_T(q^2) = \frac{G_A^2(q^2)}{g_A^2}\left[\frac{2}{3}\frac{q^2}{q^2+m_\pi^2} - \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right] + \frac{1}{3}\frac{G_M^2(q^2)}{g_A^2}\frac{q^2}{4m_p^2}$$

$$G_A(q^2) = g_A\left(\frac{\Lambda_A^2}{\Lambda_A^2+q^2}\right)^2 \qquad G_V(q^2) = g_V\left(\frac{\Lambda_V^2}{\Lambda_V^2+q^2}\right)^2 \qquad G_M(q^2) = (\mu_p - \mu_n) G_V(q^2)$$

Calculation of the products $G^{(2,0)\nu}$ x $M^{(2,0)\nu}$

i) avoids confusions regarded units; ii) makes a "correlated" calculation (with the same nuclear input parameters for both quantities); iii) improve precision in calculation through a better management of the nuclear parameters that enter both quantities; iv) separates the strong dependence on $g_A{}^4$ *,* allowing a better investigation of the uncertainties regarding this quantity

2νββ

$$[T^{2\nu}]^{-1} = G^{2\nu}(E_0, Z) \times g_A{}^4 \times |m_e c^2 \, M^{2\nu}|^2$$

$[T^{2\nu}]^{-1}$ ↓ $[yr^{-1}]$; $G^{2\nu}(E_0, Z)$ ↓ $[yr^{-1}]$; $|m_e c^2 \, M^{2\nu}|^2$ ↓ dimensionless

0νββ

$$[T^{0\nu}]^{-1} = G^{0\nu}(E_0, Z) \times g_A{}^4 \times |M^{0\nu}{}_l|^2 \times \langle\eta_l\rangle^2$$

$[T^{0\nu}]^{-1}$ ↓ $[yr^{-1}]$; $G^{0\nu}(E_0, Z)$ ↓ $[yr^{-1}]$; $|M^{0\nu}{}_l|^2$ ↓ dimensionless; $\langle\eta_l\rangle^2$ ↓ dimensionless

Input parameters entering both $G^{0\nu}$ x $M^{0\nu}$

- R_A = nuclear radius

$|M^{0\nu}| \sim R_A$, but this is compensated with $G^{0\nu} \sim 1/ R_A{}^2$ such that these quantities get the above dimensions

R_A also enters $G^{0\nu}$ through the electron w.f. which are calculated separately for the nuclear interior ($r < R_A$) and for the asymptotic part ($r > R_A$), matching them through the continuity condition at the nuclear surface. This dependence is much smaller.

For *0νββ the product* $G^{0\nu}$ x $M^{0\nu}$ *is very small affected by the choice of* r_0 .

- $<E_N>$ = average energy : enters both in *G* and *M*

The dependence of these quantities on $< E_N>$ is not big, but for consistency the same value should be used in the calculation of both.

- g_A *brings the most large uncertainty in* $0\nu\beta\beta$ *calculations, and now there is an open issue what is the "correct" value of* g_A *(quenched or not) in the case of this decay mode.*
  Values from 0.5-0.9(ShM) to 1.27 (unquenched) may induce very large uncertainties in predictions of lifetimes and values of BSM parameters (up to a factor of ~41!) much more than uncertainties from other sources.

Define products of $G^{(2,0)\nu} \times |M^{(2,0)\nu}|^2$ and calculate them at once, with the same nuclear parameters and, even better, using a consensus on the nuclear approximations to be taken into account in calculation as: FNS, SRC, higher order terms in nuclear currents expression, etc.

$2\nu\beta\beta$ $\quad P^{2\nu} = G^{2\nu} \times |m_e c^2 \; M^{2\nu}|^2$ $\quad$ *dimension=* $[yr^{-1}]$

$$[T^{2\nu}]^{-1} = (g^{2\nu}_{A,eff})^4 \times P^{2\nu}$$

$0\nu\beta\beta$ $\quad P^{2\nu}_l = G^{0\nu} \times |M^{0\nu}_l|^2$, $\quad$ l = mechanism; $\quad$ *dimension=* $[yr^{-1}]$

$$[T^{0\nu}]^{-1} = (g^{0\nu}_{A,eff})^4 \times P^{2\nu}_l \times <\eta_l>^2$$

Lifetimes Predictions

2νββ $[T^{2\nu}]_n = [(g^{2\nu}{}_{A,eff})_m{}^4 / (g^{2\nu}{}_{A,eff})_n{}^4] \times [\mathscr{P}^{2\nu}{}_m / \mathscr{P}^{2\nu}{}_n] \times [T^{2\nu}]_m$

0νββ $[T^{0\nu}]_n = [(g^{0\nu}{}_{A,eff})_m{}^4 / (g^{0\nu}{}_{A,eff})_n{}^4] \times [\mathscr{P}^{0\nu}{}_m / \mathscr{P}^{0\nu}{}_n] \times [T^{0\nu}]_m$

ratios of $g^{(2,0)\nu}{}_{A,eff}$ and of $\mathscr{P}^{(2,0)\nu}$ *instead of single values of these quantities can reduce uncertainties*

Table I. Products of *G x M* for *2νββ and 0νββ*

Nucleus	$T^{2\nu}$ *[yr]*	$\mathcal{P}^{2\nu}$ [yr^{-1}]	$T^{0\nu}$ *[yr]*	$\mathcal{P}_{\nu}^{0\nu}$ [yr^{-1}]
^{48}Ca	6.40 x 10^{19} [1]	123.81 x 10^{-21} $g_{A,eff}$ = 0.65/0.71th[8]	> 2.0 x 10^{22} [1]	16.13 x 10^{-15}
^{76}Ge	1.92 x 10^{21} [2]	5.16 x 10^{-21} $g_{A,eff}$ = 0.56/0.60th[7]	> 5.3 x 10^{25} [3]	23.94 x 10^{-15}
^{82}Se	0.92 x 10^{20} [2]	186.62 x 10^{-21} $g_{A,eff}$ = 0.49/0.60th[7]	> 3.6 x 10^{23} [9]	83.99 x 10^{-15}
^{130}Te	8.20 x 10^{20} [4]	25.26 x 10^{-21} $g_{A,eff}$ = 0.47/0.57th[7]	> 4.0 x 10^{24} [5]	64.00 x 10^{-15}
^{136}Xe	2.16 x 10^{21}[1]	20.30 x 10^{-21} $g_{A,eff}$ = 0.45/0.39th[7]	> 1.1 x 10^{25} [6]	44.11 x 10^{-15}

[1] NEMO3, PRD **93**(2016); [2] Patrignani, C. et al. (PDG), China Phys. C **40**(2016);
[3] *GERDA II,* Nature, **544**(2017)*;* [4] CUORE, EPJ C **77**(2017) ; [5] CUORE, PRL**115** (2015);
[6] EXO, *Nature.* ***510***, *229 (2014)*; [7]Caurier, PLB**71**(2012); [8]Iwata et al., PRL**116**(2016);
[9] V.I. Tretyak, NEMO3, AIP Conf.Proc. 1417,125 (2011).

Lorentz violation in weak decays

- LV can also be investigated in β and ββ decays
- The general framework characterizing LV is the Standard Model Extension (SME)
- In minimal SME (operators dimension ≤ 4) there are operators that couples to ν_s and affect ν flavor oscillations, ν velocity or ν phase spaces (β, ββ decays)
- There is a q-independent operator (countershaded operator), that doesn't affect ν oscillations, and hence can not be detected in LBL experiments
- The corresponding coefficient has 4 components (one time-like, $(å^{(3)}{}_{of})_{00}$ and 3 space-like); a non-zero value of $å^{(3)}{}_{of})_{00}$ would produce small deviations in the shape of the electrons spectrum.

In *2νββ* - the electron energy sum spectrum may receive a correction that is maximized at a well-defined energy, depending of the isotope

- the one electron spectra (angular correlation) can be modified (for experiments with tracking systems that can reconstruct the direction of the two emitted electrons.

In *0νββ* LV Majorana couplings modify the neutrino propagator, introducing novel effects in 0*νββ:* there is a charge-conjugation-preserving operator that can trigger *0νββ* even if the Majorana m_ν is negligible → lower bounds on the half life $T^{0\nu}{}_{1/2}$ can also be used to constrain the relevant coefficients for LV

- Until now, the most precise tests for LV involving ν_s are perform in ν oscillation experiments.
- Now, deviations due to LV are also investigated in DBD experiments, like EXO and NEMO3.

The coupling of the ν to the countershaded operator modifies the neutrino momentum from the standard expression:

$q^{\alpha} = (\omega, \mathbf{q}) \rightarrow q^{\alpha} = (\omega, \mathbf{q} + \mathbf{a}^{(3)}_{of} + \mathring{a}^{(3)}_{of}\,\mathbf{q})$ J.S. Diaz, PRD89(2014)

This deviation modifies the *2νββ transition amplitude and the neutrino dispersion relation.*
The decay rate can be written as a sum of the standard term and a perturbation due to LVββ

$\Gamma^{(2\nu)} = \Gamma_0^{(2\nu)} + d\Gamma^{(2\nu)}$

$\Gamma_0^{(2\nu)} = G_0^{2\nu}(E_0, Z) \times g_A^4 \times |m_e c^2\, M^{2\nu}|^2$

$d\Gamma^{(2\nu)} = dG^{2\nu}(E_0, Z) \times g_A^4 \times |m_e c^2\, M^{2\nu}|^2$

$G_0^{2\nu} = C \int_0^Q d\varepsilon_1 F(Z, \varepsilon_1) [\varepsilon_1(\varepsilon_1 + 2)]^{1/2} (\varepsilon_1 + 1) \int_0^{Q-\varepsilon 1} d\varepsilon_2 F(Z, \varepsilon_2) [\varepsilon_2(\varepsilon_2 + 2)]^{1/2} (\varepsilon_2 + 1)(Q - \varepsilon_1 - \varepsilon_2)^5$

$dG^{2\nu} = 10\mathring{a}^{(3)}_{of}\, C \int_0^Q d\varepsilon_1 F(Z, \varepsilon_1) [\varepsilon_1(\varepsilon_1 + 2)]^{1/2} (\varepsilon_1 + 1) \int_0^{Q-\varepsilon 1} d\varepsilon_2 F(Z, \varepsilon_2) [\varepsilon_2(\varepsilon_2 + 2)]^{1/2} (\varepsilon_2 + 1)(Q - \varepsilon_1 - \varepsilon_2)^4$

$C = (G_F^4 (\cos_\Theta)^4 m_e)/240\pi^7$ $t_{1,2} = \varepsilon_{1,2} - 1;$

J.S. Diaz, PRD**89**(2014),
EXO collab., arXiv:1601.07266v2[nucl-ex]

$G_0^{2\nu}$, $dG^{2\nu}$ can be calculated in different approximations:

- $F(Z, \varepsilon) = (2\pi y)[1 - \exp(-2\pi y)]^{-1}$, $y = \pm\alpha Z\varepsilon/q$, Primakoff&Rosen, RPP**22**(1959)
- $F(Z, \varepsilon) = 4(2qR_A)^{2(\gamma-1)} |\Gamma(\gamma + iy)|^2 \exp(\pi y) |\Gamma(2\gamma + 1)|^{-2}$ Suhonen&Civitarese, PR**301**(1998)
- using exact electron functions obtained by solving Dirac equations Iachello,PRC**85**2012; Stoica, PRC**88**(2013), RRP63(2015)

Nucleus	$Q_{\beta\beta}$[MeV]	$G_0^{2\nu}/C$	$dG^{2\nu}/10\,\mathring{a}^{(3)}_{of}\,C$	$dG^{2\nu}/10\mathring{a}^{(3)}_{of}\,G_0^{2\nu}$
^{48}Ca	4.267	$3.85{\bullet}10^6$	$8.55{\bullet}10^5$	0.222
^{76}Ge	2.039	$1.28{\bullet}10^4$	$0.31{\bullet}10^4$	0.242
^{82}Se	2.996	$3.83{\bullet}10^5$	$1.14\ {\bullet}10^5$	0.297
^{100}Mo	3.034	$7.86{\bullet}10^5$	$2.30{\bullet}10^5$	0.293
^{136}Xe	2.458	$2.57{\bullet}10^5$	$0.74{\bullet}10^5$	0.288

EXO, 2016: $-2.65 \times 10^{-5} < \mathring{a}^{(3)}_{of} < 7.60 \times 10^{-6}$ GeV (90% CL) (arXiv:1601.07266v2[nucl-ex]

Conclusions

•DBD is a nuclear rare decay process with a broad potential to investigate BSM physics (LNV, limits for neutrino mass parameters, limits for other BSM parameters related to specific possible mechanisms of occurrence of $0\nu\beta\beta$: Majoron, SUSY exchange, LV, RH components of WI, etc.)

•Calculation of NME & PSF are still subject of significant uncertainties, one most important being the axial-vector constant g_A

•Calculation of products G x M would have the following advantages:

-*avoids confusions regarded units*
-*makes a "correlated" calculation concerning the nuclear parameters*
-*separates the strong dependence on* g_A^4 , allowing a detailed study on uncertainties regarding this quantity

•Lorentz violation can also be tested in DBD

- in $2\nu\beta\beta$: one can derive limits on the isotropic coefficient that control the distortions in electron spectra due to LV
- in $0\nu\beta\beta$: one can derive limits for the strengths of SME LV operators that can lead to the occurrence of this decay mode when $<m_{\beta\beta}>$ vanishes