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Electron antineutrino spectrum of nuclear reactor and forbidden beta decays

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Outline

1. Essences of calculation the reactor antineutrino signal

1.1 Creating antineutrinos in the nuclear reactors

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2.1 The ‘ab initio’ summation method

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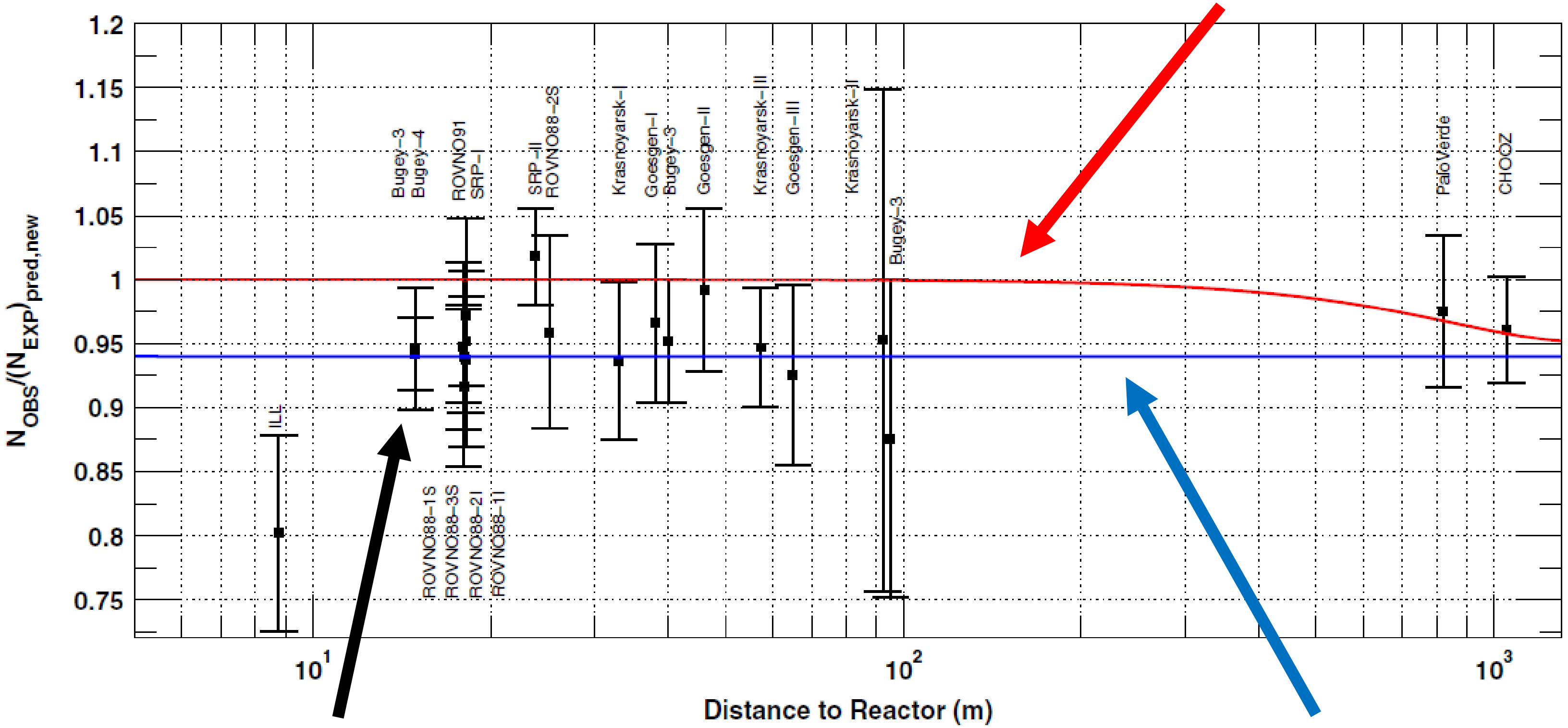
3. Conclusions

Reactor anomaly problem: 6 % antineutrino missing.

*Mueller et al., Phys. Rev. C **83**, 054615 (2011)*
*Huber, Phys. Rev. C **84**, 024617 (2011)*

Theoretical prediction; standard three-active neutrino mixing



Experimental mean averaged N_{OBS}/N_{EXP} =0.943±0.023

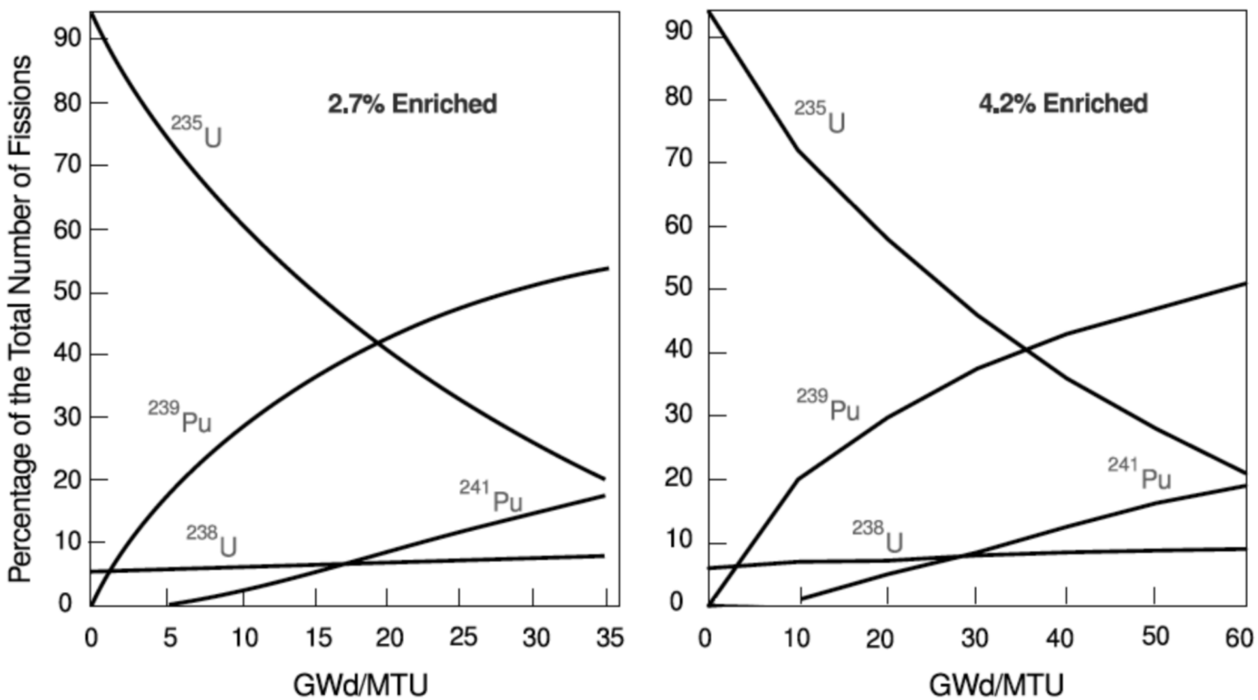
Theoretical prediction including a new sterille neutrino, $|\Delta m^2| >> 1\ eV^2$, $\sin^2(2\vartheta)=0.12$

*Mention et al., Phys. Rev. D**83**, 073006(2011)*

1. Essences of calculation the reactor antineutrino signal

1.1 Creating antineutrinos in nuclear reactors

In power reactors 99.9 % of the power comes from the fission of ^{235}U, ^{238}U, ^{239}Pu and ^{241}Pu.



Antineutrinos are creating in the β-decays of neutron-rich fissions products of ^{235}U, ^{238}U, ^{239}Pu and ^{241}Pu :

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{\;\;A}_{Z+1}X_{N-1} + e^{-} + \overline{\nu} $$

$$ S(E_\nu) = \sum_i f_i \frac{dN_i}{dE_\nu} $$

Total antineutrino spectra

Number of fissions from i-th actinide

Antineutrino fission spectra of i –th actinide

How to obtain the f_i ?

$$ f_i = \frac{P_{th}\ \ p_i}{Q_i} $$

The energy released in the fission of i-th actinide.

The power fraction of i-th actinide.

Isotope	Q_i (MeV)
^{235}U	202.36 ± 0.26
^{238}U	205.99 ± 0.52
^{239}Pu	211.12 ± 0.34
^{241}Pu	214.26 ± 0.33

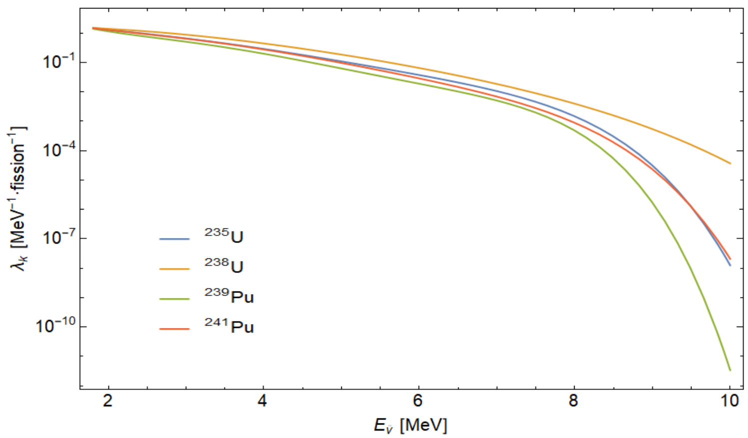
T.A. Mueller et al., Phys. Rev C **83**, 054615 (2011)

Type of reactor	^{235}U	^{238}U	^{239}Pu	^{241}Pu
PWR BWR LWGR GCR	0.560	0.080	0.300	0.060
PHWR	0.543	0.024	0.411	0.022
MOX	0.000	0.081	0.708	0.212

Many prescriptions for the antineutrino fission spectra of i –th actinide $\frac{dN_i}{dE_\nu}$ exist.

According to Mueller *et al.* the spectra of all four contributing isotopes are given in terms of the exponential of a polynomial of order 5:

$$\frac{dN_i}{dE_{\bar{\nu}}} = Exp\left(\sum_i a_p^i \; E_{\bar{\nu}}^{p-i}\right)$$



Isotope	a_1	a_2	a_3	a_4	a_5	a_6
^{235}U	3.217	-3.111	1.395	-0.369	0.04445	-0.002053
^{238}U	0.4833	0.1927	-0.1283	-0.006762	0.002233	-0.0001536
^{239}Pu	6.413	-7.432	3.535	-0.882	0.1025	-0.00455
^{241}Pu	3.251	-3.204	1.428	-0.3675	0.04254	-0.001896

T.A. Mueller et al., Phys. Rev C **83**, 054615 (2011)

How is this formula correct? …

1.2 Propagation of the antineutrinos in the space

1. The flux of antineutrinos decreases with the distance from the reactor *d* as:

$$\Omega_{\mathrm{d}} = \frac{\Omega_{\mathrm{init}}}{4\pi \mathrm{d}^2}$$

2. The electron antineutrinos oscillate and the survival probability can be write as:

$$\mathrm{P_{ee}} = 1 - \sin^2 2\theta_{13}\left(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right) \\ -\cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21},$$

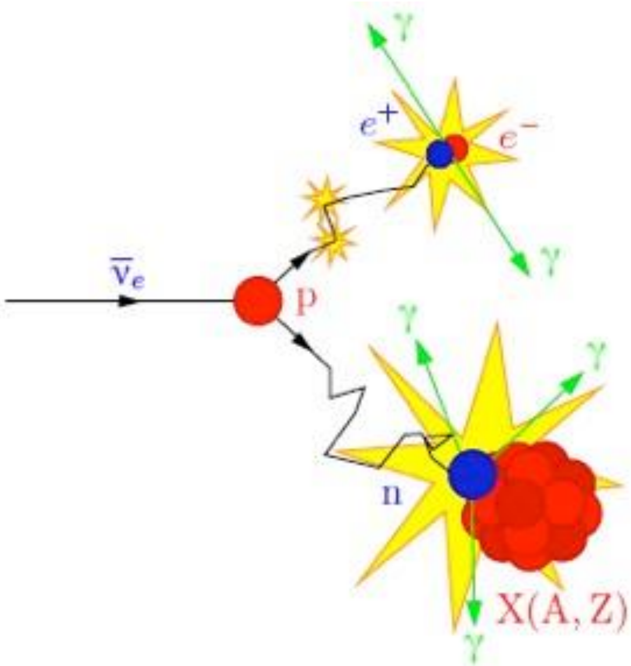
$$\Delta_{\mathrm{ij}} = \Delta \mathrm{m}_{\mathrm{ij}}^2\, \mathrm{d}/4\, \mathrm{E}_{\bar{\nu}}$$

A. Fengpeng et al., J. Phys. G: Nucl. Part. Phys. **43** (2016) 030401

1.3 Detection of the antineutrinos

Inverse beta decay (IBD)

$$\bar{\nu}_e + p \rightarrow e^+ + n$$



The cross section of the IBD can be parametrized as

$$\boldsymbol{\sigma_{IBD}(E_{\bar{\nu}}) = 10^{-43} cm^2 p_e E_e E_{\bar{\nu}}^{-0.07056+0.02018 ln E_{\bar{\nu}} - 0.001953 ln^3 E_{\bar{\nu}}}}$$

A. Strumia and F. Vissani, Phys. Letters B **564**, 42 (2003)

The final formula for the antineutrino reactor signal

$$N_{tot} = \varepsilon N_p \tau \sum_{i=1}^{N_{reactor}} \frac{P_{th}^i}{4\pi d_i^2} \langle LF_i \rangle \int dE_{\bar{\nu}} \sum_{k=1}^{4} \frac{p_k}{Q_k} \frac{dN_{\bar{\nu}}}{dE_{\bar{\nu}}} P_{ee}(E_{\bar{\nu}}, d_i) \sigma_{IBD}(E_{\bar{\nu}})$$

- Reactor
- Propagation
- Detection

For the reference and comparison reasons the signal is calculated in Terrestrial Neutrino Units (TNU)

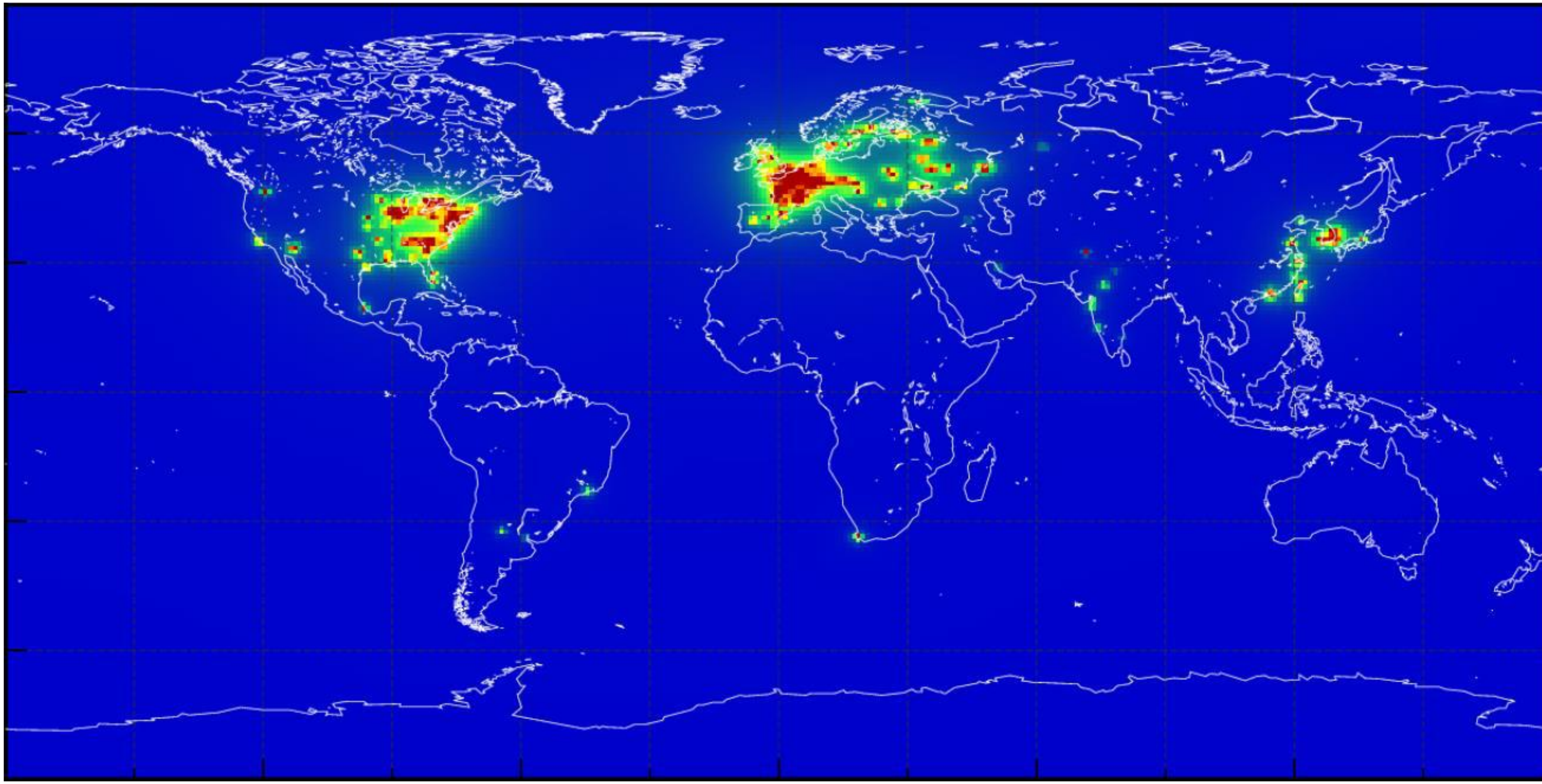
- Detector effeciency $\boldsymbol{\varepsilon = 1}$
- Number of free protons $\boldsymbol{N_p = 10^{32}}$ ($=$ 1 kton of liquid scintilator detector)
- Time of measurement $\boldsymbol{\tau = 3.15 \times 10^7 s}$ (1 year)

1 TNU

=

1 detected neutrino by 1 kton liquid scintilator detector with effeciency $\boldsymbol{\varepsilon = 1}$

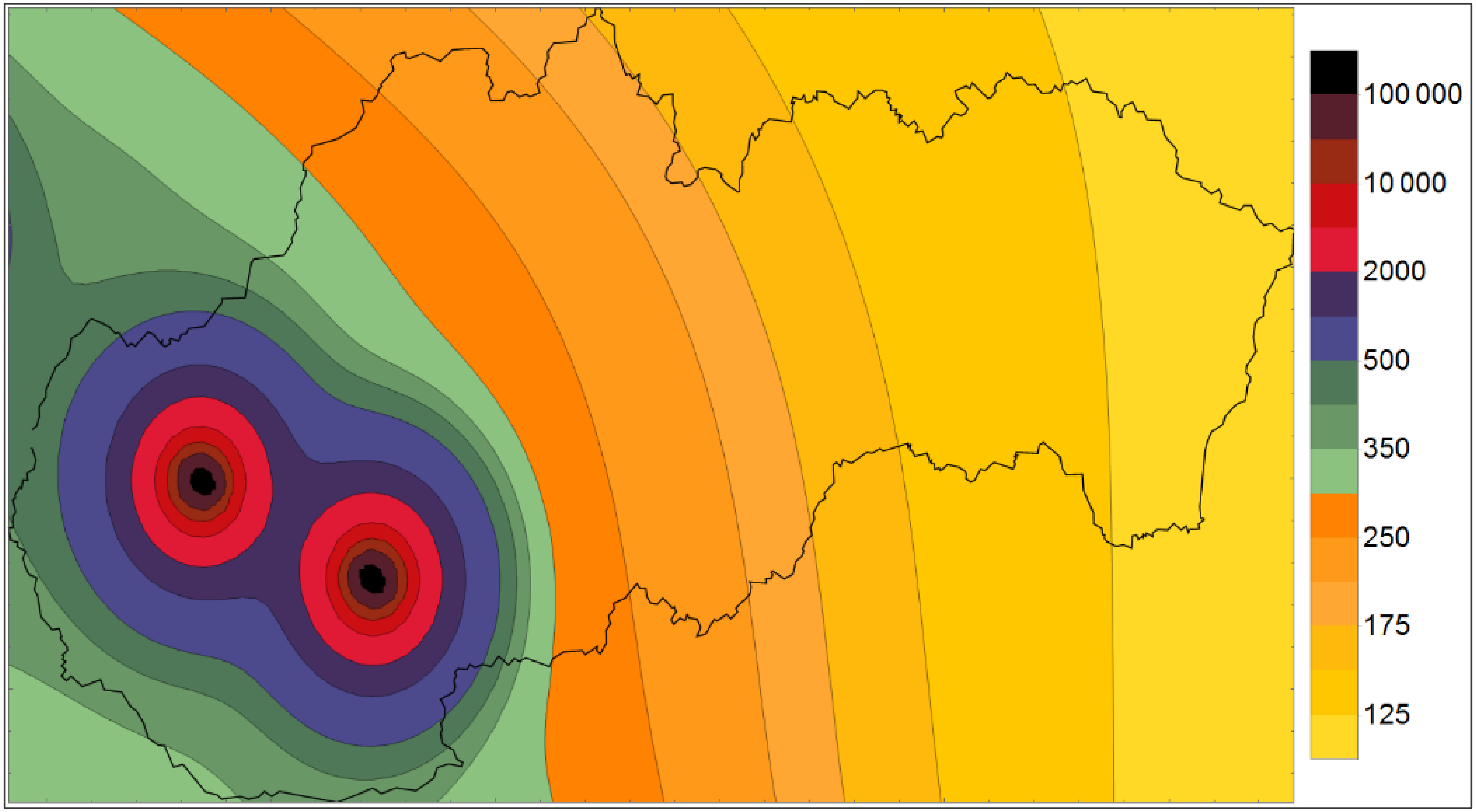
World reactor antineutrino signal map (2013)



(LER antineutrinos only)

M.Baldoncini et al, Phys.Rev.D91065002(2015)

Reactor antineutrino signal map of Slovakia (2015)



Made by P. Kerényi (Comenius University, Bratislava)

2. Fission antineutrino spectrum

2.1. The ‘ab initio’ summation method

In the ‘ab initio’ approach the aggregate fission antineutrino spectrum is determined by summing the contributions of all β-decay branches of all fission fragments

$$\frac{dN_i}{dE_{\bar{\nu}}} = \Sigma_n Y_n(Z,A,t)\Sigma_{n,i} b_{n,i}(E_0^i) P_{\bar{\nu}}(E_{\bar{\nu}}, E_0^i, Z)$$

$Y_n(Z,A,t)$ Number of beta decays of the fragment Z, A at given time t (fission yelds)

$b_{n,i}(E_0^i)$ Branching ratios with endpoint energies E_0^i $\quad \sum_{n,i} b_{n,i}(E_0^i) = 1$

$P_{\bar{\nu}}(E_{\bar{\nu}}, E_0^i, Z)$ Normalized anti-neutrino spectra shape for the i-th branch

n characterize ground and isomeric state

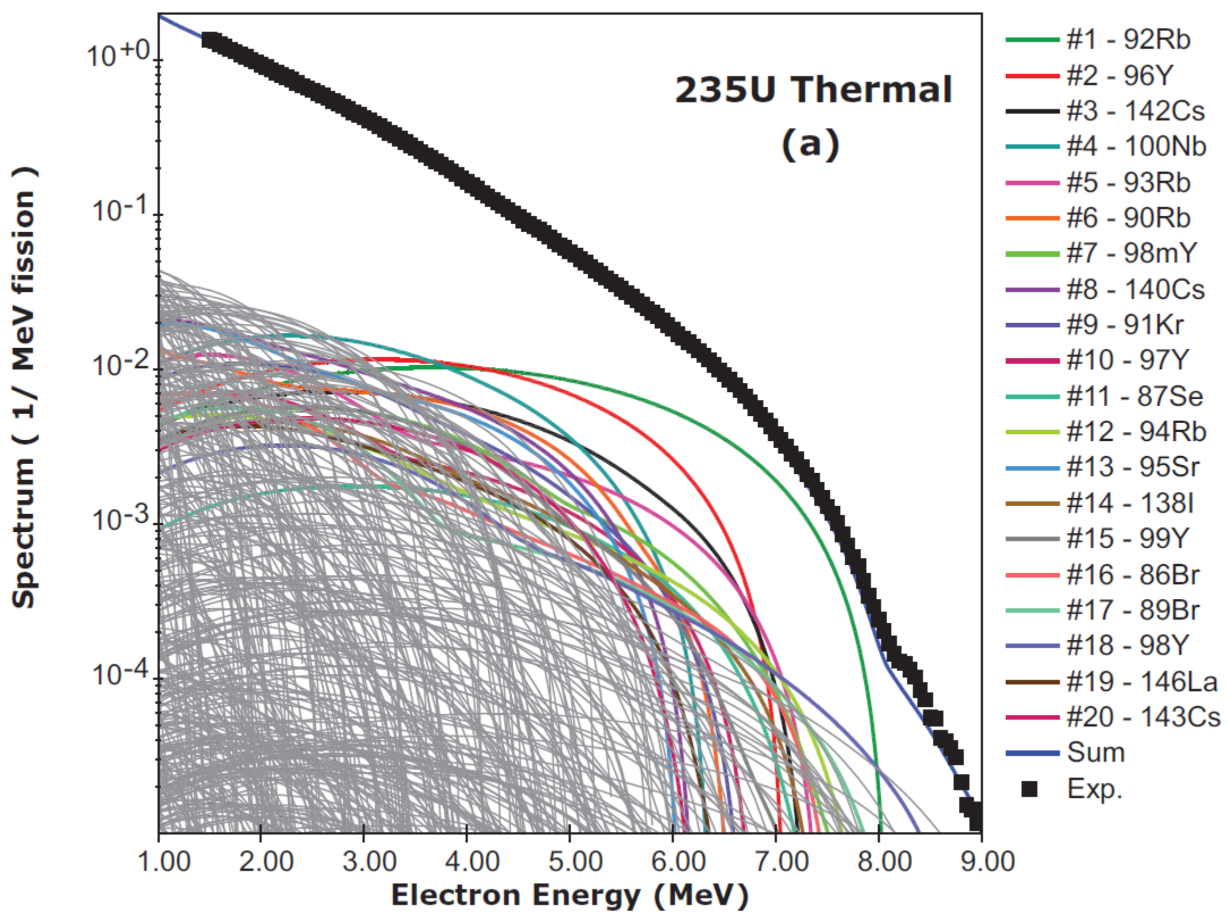
In applying the summation method several sources of uncertainty arise:

The fission yields Y_n have been evaluated by several international database groups, but for many important fragments the yields involve large uncertainties.

The branching ratios $b_{n,i}$ are also not known for all fragments, and nor are the quantum numbers (spins and parity) of all of the initial and final states.

The shape of the β decay spectrum $P_{\bar{\nu}}(E_{\bar{\nu}}, E_0^i, Z)$ is well known for allowed transitions. However $\sim$ 30% of the transitions making up the aggregate spectra are known to be so-called first forbidden transitions and involve nuclear structure dependent combinations of several more complicated operators.

A. A. Sonzogni et al., PHYS. REV. C **91**, 011301(R) (2015)



Need to sum beta spectra of around 800 fission products

The AB INITIO method describes the spectra with only 10-20% of accuracy

But it is applied for the ^{238}U isotope.

2.2 The electron spectrum conversion method.

The second method of determining the $dN_i / dE_{\bar{\nu}}$ spectra begins with the experimentally measured aggregate electron spectrum of each actinide i.

$$\frac{dN_{ei}}{dE_e} = \sum_i a_i P_{\bar{\nu}}^i (E_e, E_0^i, Z)$$

The measured spectra are fitted by the sum of the 30-40 single beta spectra with the virtual endpoint energies E_0^i .

The antineutrino spectra one obtain with just replacing the electron energy by $E_0^i - E_{\bar{\nu}}$

$$\frac{dN_{\bar{\nu}i}}{dE_{\bar{\nu}}} = \sum_i a_i P_{\bar{\nu}}^i (E_0^i - E_{\bar{\nu}}, E_0^i, Z)$$

The electron spectrum for thermal neutron fission of ^{235}U, ^{239}Pu and ^{241}Pu were measured at ILL, Grenoble, France in 1980's.

K. Schreckenbach et al., Phys. Lett. B **160**, 325 (1985).

Beta –spectra, shape factors and finite-size corrections

$$P_{\bar{\nu}}^{i}(E_e, E_0^i, Z) = N \;\; p_e E_e (E_0 - E_e)^2 F_0(E_e, Z, A) \;\; C(E_e)(1+\delta(E_e, Z, A))$$

Normalization coefficient

Fermi function

Shape factors, for the allowed transition $C(E_e)=1$

Finite – size, radiative, and weak-magnetism corrections

30 % of the neutrino flux arises from the first forbidden beta transitions.

Brief history of the reactor neutrino spectrum determination:

1. First `modern' evaluations were done in late 1970 and early 1980

(Davis et al. 1979, Vogel et al. 1981, Klapdor & Metzinger 1982)

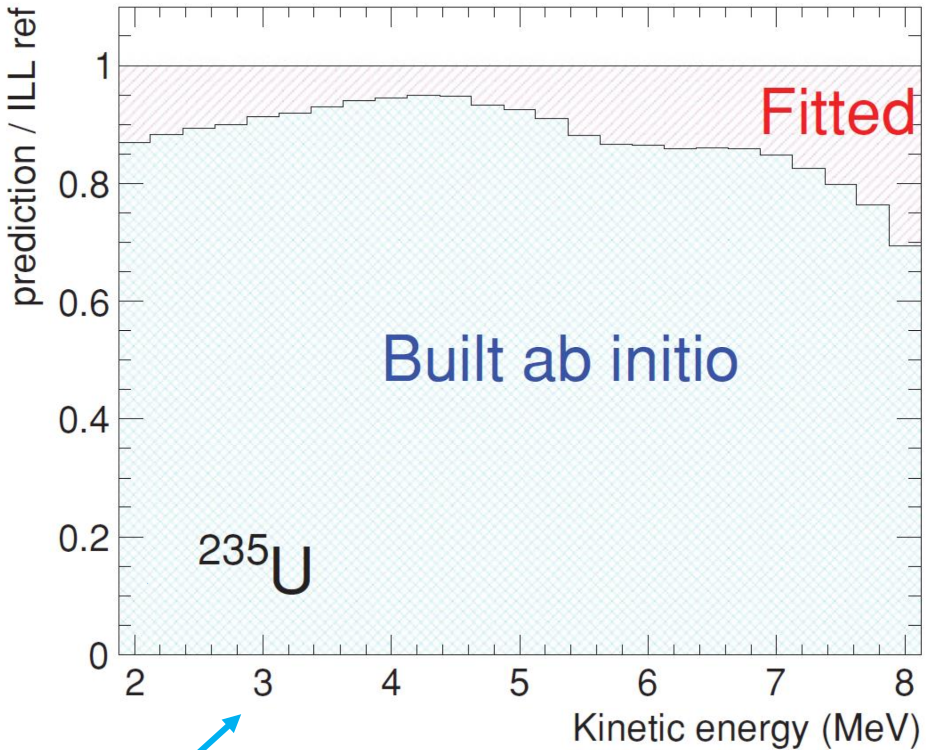
2. During the 1980-1990 a series of measurements of the **electron** spectra associated with the fission of ^{235}U, ^{239}Pu and ^{241}Pu were performed at ILL Grenoble by Schreckenbach et al. These were **converted** into the electron antineutrino spectra by the authors.

3. New evaluation (Mueller et al. 2011) uses a combination of the ab initio approach with updated experimental data and the input from the converted electron. **This results in the upward shift by ~3% of the reactor flux.**

Huber 2011 obtain also the ~3% shift using the electron conversion method with corrections

Inspired by slide of P. Vogel

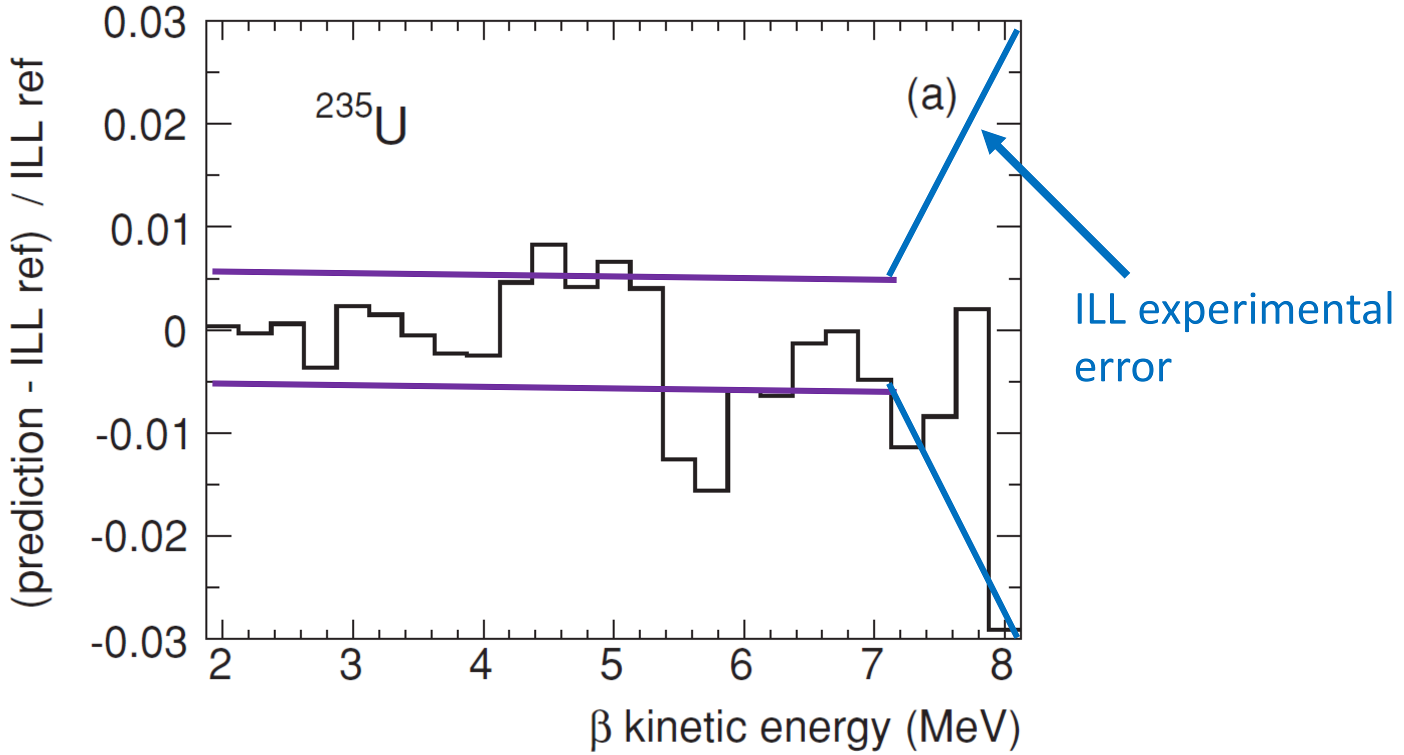
Brief overview of Mueller et al., Phys. Rev. C 83 (2011)



Fitted by 5 virtual beta allowed transitions with Z=46

All forbidden transitions are calculate as the unique forbidden – this can affect results a lot!

The obtained electron spectra do not correspond to the ILL experimental one within the experimental error!!!



Brief overview of Huber, Phys. Rev. C 84, 024617 (2011)

The electron spectrum conversion method is used but adding more precise finite – size, radiative, and weak-magnetism corrections

$$P_{\bar{\nu}}^{i}(E_e, E_0^i, Z) = N \;\; p_e E_e (E_0 - E_e)^2 F_0(E_e, Z, A) \;\; C(E_e)(1 + \delta(E_e, Z, A))$$

But no forbidden transitions included!

A.C. Hayes, PRL 112 (2014)

“Given the present lack of detailed knowledge of the structure of the forbidden transitions, it is not possible to convert the measured aggregate fission beta spectra to antineutrino spectra to the accuracy needed to infer an anomaly.”

$\rightarrow$

Reactor Neutrino Spectra Anna Hayes and Peter Vogel:
Ann.Rev.Nucl.Part.Sci. 66 (2016) 219-244
“we estimate that the uncertainty in antineutrino spectra derived by the conversion method are about 5%. The uncertainties in the summation method are considerably worse and at least in the upper part of the antineutrino spectrum are probably up to the 20% level. We emphasize that these are our subjective estimates. They are based on educated guesses and they do not represent statistical variances.”

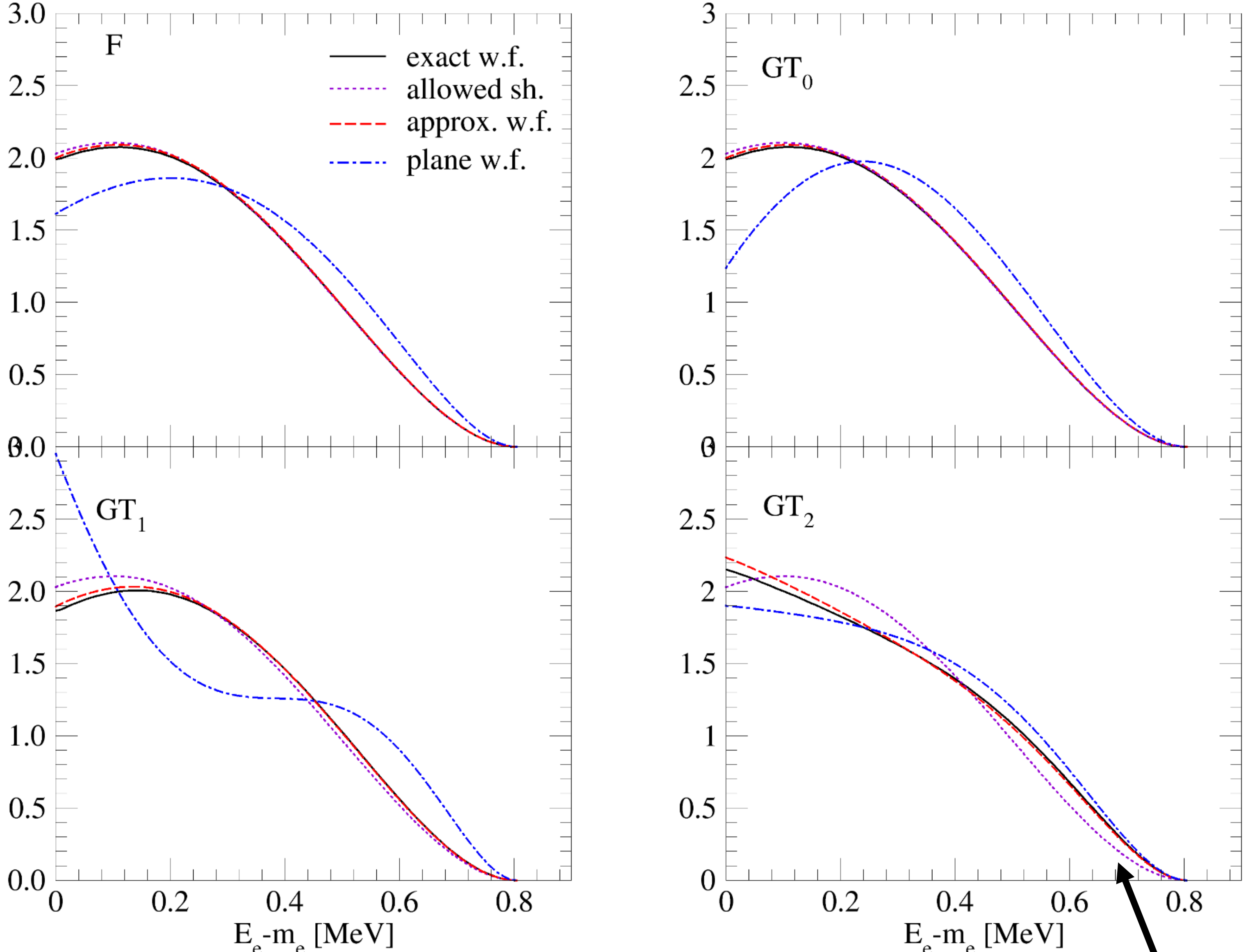
Shape factors for the first forbidden transitions

Operator	Plane wave shape factor $C_i(E)$ (no Coulomb)	Exact relativistic $C_i(E)$
$GT_0\left(\{r\sigma\}_0\right)$	$\frac{R^2}{9}\left(p_\nu^2+p_e^2+2\frac{p_\nu^2 p_e^2}{E_\nu E_e}\right)$	$\frac{R^2}{9}\left(p_\nu^2+p_e^2\tilde{F}_{p_{1/2}}+2p_e p_\nu\frac{p_\nu}{E_\nu}\tilde{F}_{sp_{1/2}}\right)$
$F\left(r\tau\right)$	$\frac{R^2}{9}\left(p_\nu^2+p_e^2+\frac{2}{3}\frac{p_\nu^2 p_e^2}{E_\nu E_e}\right)$	$\frac{R^2}{9}\left(p_\nu^2+\frac{1}{3}p_e^2\tilde{F}_{p_{1/2}}+\frac{2}{3}p_e^2\tilde{F}_{p_{3/2}}+\frac{2}{3}p_\nu p_e\frac{p_\nu}{E_\nu}\tilde{F}_{sp_{1/2}}\right)$
$GT_1\left(\{r\sigma\}_1\right)$	$\frac{R^2}{9}\left(p_\nu^2+p_e^2-\frac{4}{3}\frac{p_\nu^2 p_e^2}{E_\nu E_e}\right)$	$\frac{R^2}{9}\left(p_\nu^2+\frac{2}{3}p_e^2\tilde{F}_{p_{1/2}}+\frac{1}{3}p_e^2\tilde{F}_{p_{3/2}}-\frac{4}{3}p_\nu p_e\frac{p_\nu}{E_\nu}\tilde{F}_{sp_{1/2}}\right)$
$GT_2\left(\{r\sigma\}_2\right)$	$\frac{R^2}{9}\left(p_\nu^2+p_e^2\right)$	$\frac{R^2}{9}\left(p_\nu^2+p_e^2\tilde{F}_{p_{3/2}}\right)$
$J_V\ (\approx E_0\ r\ \tau/3)$	1	1
$\rho_A\left(\approx\lambda\ E_0\ r\left\{r\sigma\right\}_0\right)$	1	1

$$
\begin{aligned}
\frac{d\Gamma}{dE_e} = G_\beta^2\frac{1}{2\pi^3}p_e E_e p_\nu E_\nu F_0(E_e,Z_f)\ \times\ &\Big(\ F_{p_{3/2}}(E_e,R)\frac{1}{9}(p_e R)^2 M_{p_{3/2}} + F_{p_{1/2}}(E_e,R)\frac{1}{9}(p_e R)^2 M_{p_{1/2}} \\
&+\ F_{s_{1/2}}(E_e,R)\left[\frac{1}{9}(p_\nu R)^2 M_{s_{1/2}} + M_{J_V,\rho_A} + 2\frac{p_\nu}{E_\nu}\frac{1}{3}(p_\nu R)M_{s_{1/2},J_V,\rho_A}\right] \\
&+\ F_{sp_{1/2}}(E_e,R)\left[2\frac{p_\nu}{E_\nu}\frac{1}{9}p_e p_\nu R^2 M_{sp} - \frac{2}{3}(p_e R)M_{sp,J_V,\rho_A}\right]\ \Big)
\end{aligned}
$$

The problem is that many authors use for their analysis plane wave shape factors which have incorrect form!

The difference is mainly obvious in the GT_1 matrix element



^{147}Nd

Unique first forbidden beta decay shape

We saw that spectra for the first forbidden unique beta decay differs from the allowed one, but the spectra associated with the other non-unique first forbidden nuclear matrix elements are quite similar to the allowed ones.

In fact, most of the non-unique first forbidden spectra are similar with the allowed ones.

Vladimir Tretiak [1] analyzed 38 measured first forbidden spectra [2] and compared it with allowed ones:

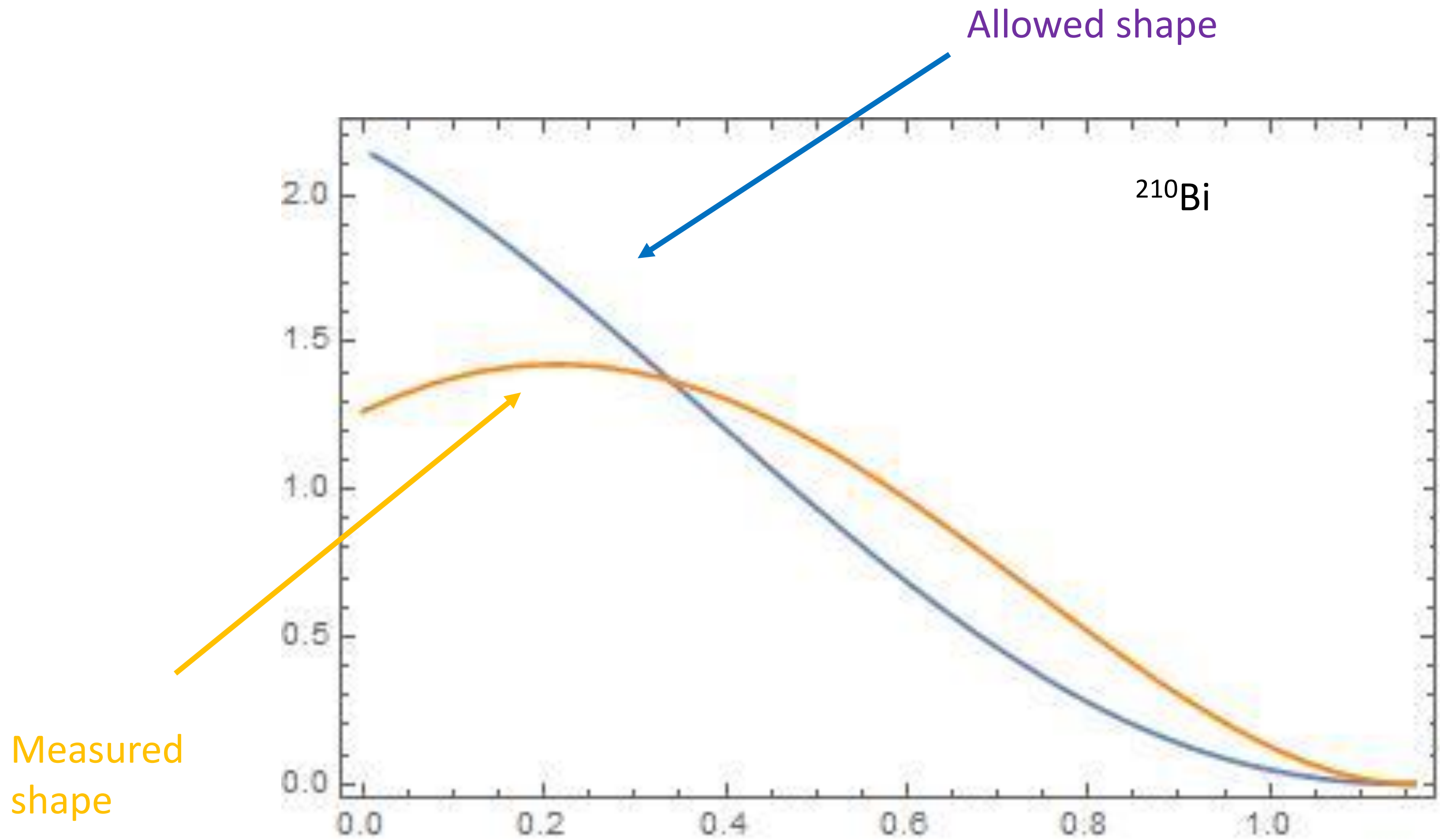
Among them

20 nuclei are allowed or very close to allowed

8 nuclei have deviation from the allowed shape in the region (5%, 10%)

10 nuclei have deviation more than 10 %

Especially ^{72}Ga, ^{115m}Cd, ^{170}Tm have deviation (20%, 30%) and ^{210}Bi (Rae) has deviation even more than 30%

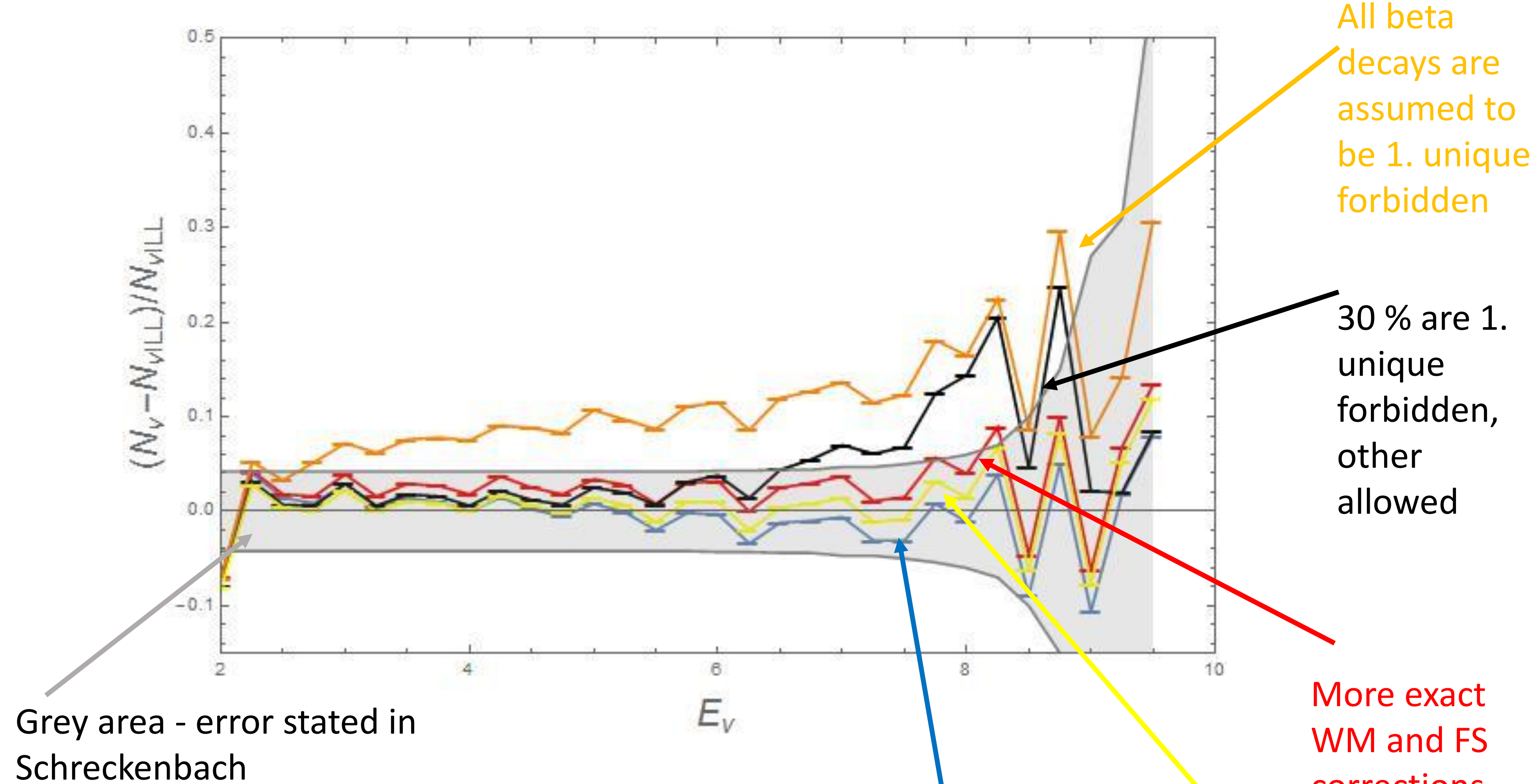


Why the spectrum is so different from the allowed one? Because of the cancelation effect between different nuclear matrix elements.

Since the shape of the non-unique first forbidden spectra depends strongly on the values of the nuclear matrix elements it is questionable if the analyzation of the neutrino spectra based on the spectra associated with the specific matrix element has a meaning.

Since we do not know the shape of the first non-unique first forbidden spectra and since many of them has allowed shape we threat the forbidden beta decays as a first unique beta decays and look at the deviations in the neutrino spectra.

A comparison between our results and Schreckenbach results $N_{\nu ILL}$



$(N^{total} - N_{\nu\,ILL}^{total} /)/(N_{\nu\,ILL}^{total})100\%$

1.6 %

1.1 %

2.7 %

1.71 %

6.35 %

K. Schreckenbach et al., Phys. Lett. B 160, 325 (1985)

Conclusions

The effects of the forbidden transitions to the fission antineutrino spectra were not be done correctly, till now. Even if they were some attempts i.e. A.C. Hayes, PRL 112 (2014).

At least we suggest that the proper shape factors, presented in our work should be used for the analysis.

From our simplified analysis it seems that including forbidden transitions has tendency to the increase the predicted number of neutrinos.

It also seems that shape factors has a subdominant role than the FS and WM corrections.

It is clear that including proper shape factor, FS, WM and radiative corrections is essential for the AB INITIO calculation, but have all these corrections sense in the case of the electron conversion method, where only virtual (not real!) beta branches are assumed? Maybe old Schreckenbach results are closer to the reality than the modern approaches...