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# *Studies with Single-Nucleon Transfer PART ONE*

*Sean J Freeman*

*In a simple way, review the aspects of nuclear structure that might influence ββ decay.*
*Describe some of the methodology of extracting occupancies from single-nucleon transfer reactions.*
*Experimental results from one-nucleon transfer reactions for the $^{100}$Mo and $^{150}$Nd systems .*

MEDEX17 - Prague

*B.P. Kay, J. P. Schiffer, C. M. Deibel and C.R. Hoffman*
*Argonne National Laboratory, Illinois, USA*

*S. J. Freeman, S.D. Blot, T. Cocolios, J.P. Enwistle,*
*A. M. Howard, S. A. McAllister, A. J. Mitchell, D. K. Sharp,*
*S.V. Szwec and J. S. Thomas*
*School of Physics and Astronomy, University of Manchester, UK*

*T. Faestermann, R. Hertenberger and H.-F. Wirth*
Maier-Leibnitz-Laboratorium der Münchner Universitäten, Garching, Germany

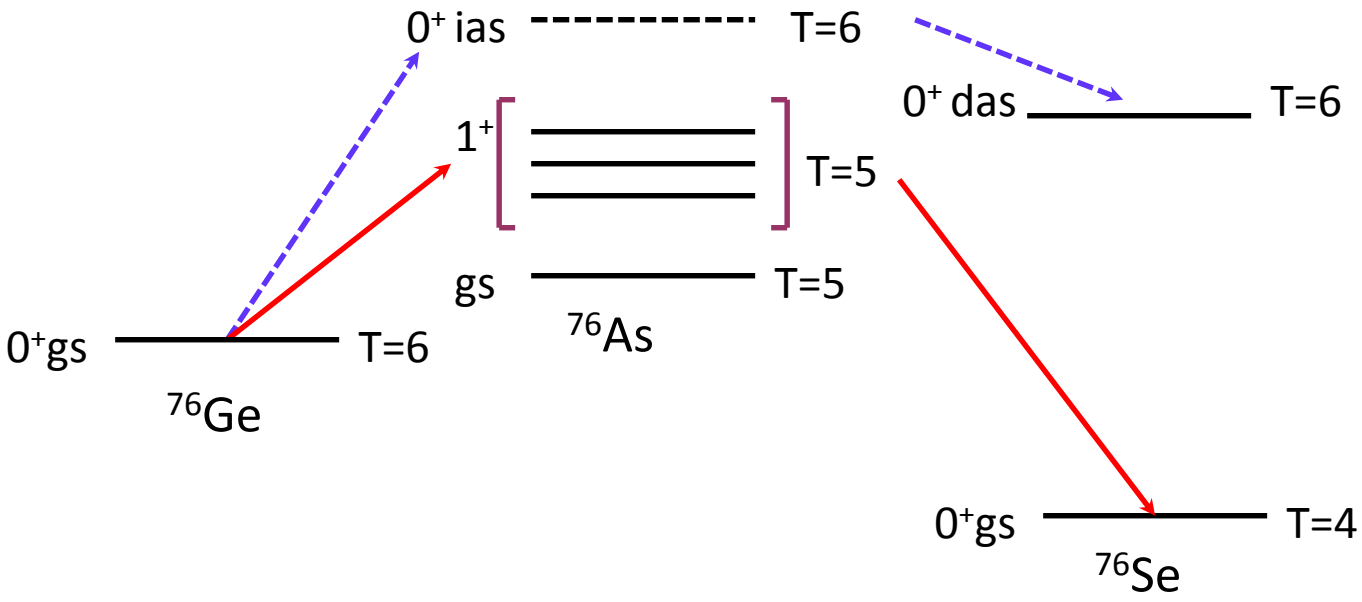
# Double beta decay with neutrinos, 2νββ

Often viewed as via virtual excitation of states in the intermediate nucleus.

GT transitions via $1^+$ states in the intermediate nucleus

Fermi part: $\Delta J=0$; super allowed if $T_i=T_f$ (can neglect)

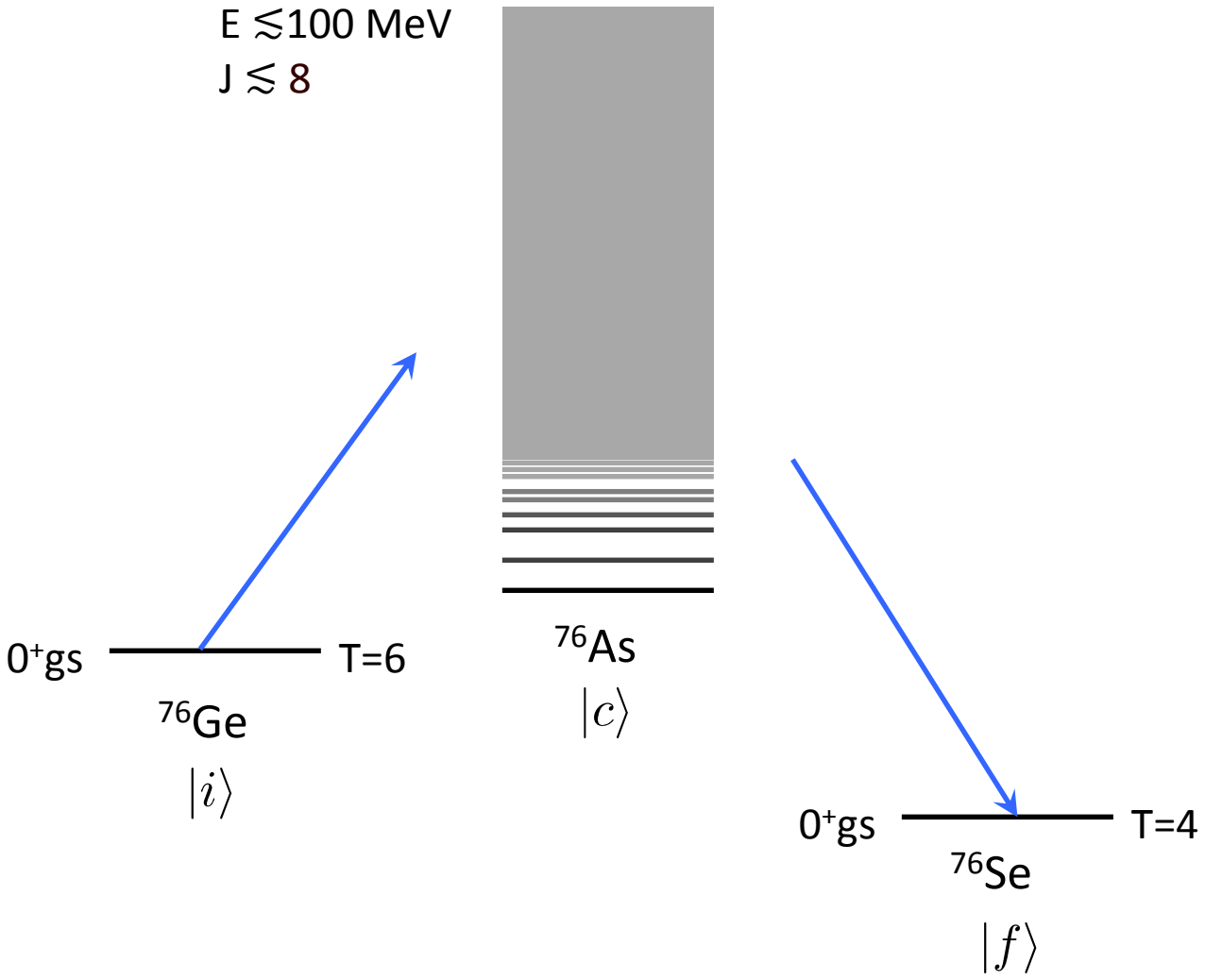
GT part: $\Delta J=\pm1,0$, except no J=0 to J=0




Effects of nuclear structure in intermediate nucleus is high, depends critically on specific locations of *low-lying* 1+ states (i.e. GT strength function) and the initial/final states.

# Neutrinoless double beta decay, 0νββ

Mediation by a virtual neutrino gives different features:



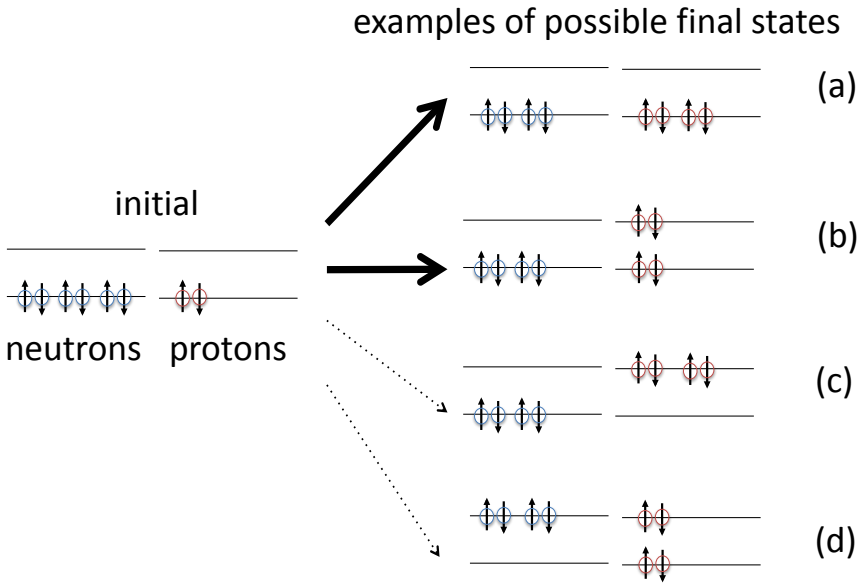

$$q \sim \hbar/r_{nn} \sim 50 - 100\mathrm{MeV/c}$$

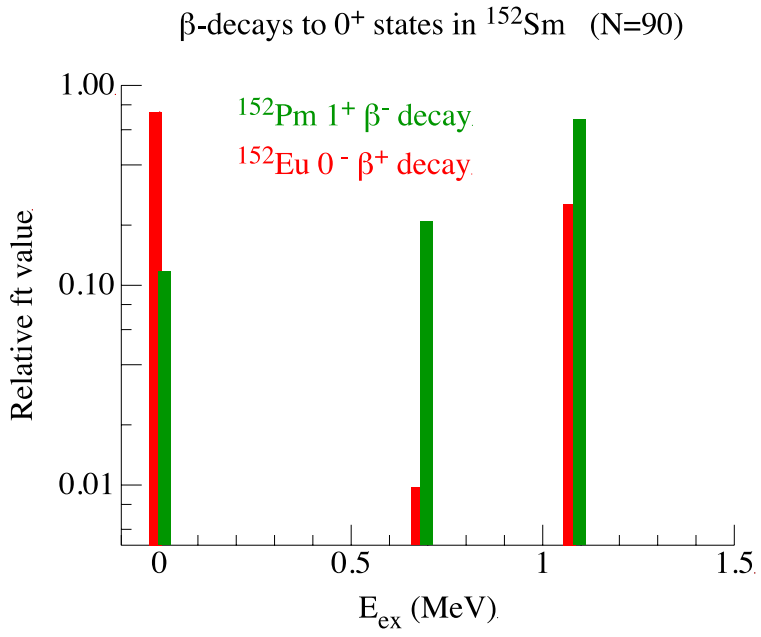
A: Energy of intermediate excited states can be large up to few tens of MeV
*(compare with few MeV for 2νββ).*

B: Angular momentum transfer is also large, up to 7-8 ħ
*(compare with 1ħ for 2νββ).*

- Less sensitive to details of intermediate nucleus - closure approximation.
- Does not seem simply related to 2νββ mode.
- Phenomenology is unlikely to work!
- Ground states of parent and daughter… *or how similar or different they are*… must matter at some level.

# Naïve caricature to illustrate:



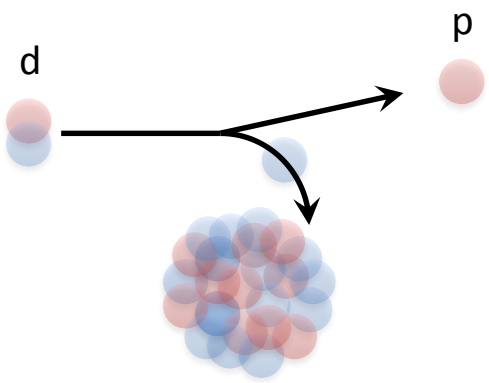

- Process might be facilitated if the parent/daughter ground states are related by simple changes of neutrons to protons.
- Significant rearrangements of nucleons other than the direct participants likely to inhibit process: e.g. very different structures or deformation.

*Real nuclei rather more complicated: independent-particle models almost never work due to strong correlations between nucleons and associated scattering can result in a state having partial occupations of single-particle levels.*

Measuring the nucleon occupancies may provide constraints for $M^{0\nu}$ calculations.

# Experimental Probe: single-nucleon transfer reactions e.g. (d,p)

Arrange experimental conditions to favour single-step transfer of a nucleon to/from target – probe single-particle degrees of freedom.




*Caricature version:*

Empty orbit: can't remove, but can add.
Full orbit: can't add, but can remove.
Partially occupied: reduced cross section.

*Complications:* due to correlations between nucleons, strength of an independent-particle model (IPM) orbit can be spread over several states.

For a particular state, define a spectroscopic overlap or factor: $$SF = |\langle \Phi_{J_B}^{M_B} | \mathcal{A}[\Phi_{J_A} \phi_j]_{J_B}^{M_B} \rangle|^2$$

*"How much does the final state look like the target plus a nucleon in a specific orbit"*

Extract from experimental cross sections by comparison with *reaction model* of cross section expected for an IPM state:

$$SF = \sigma_{\mathrm{expt}} / \sigma_{\mathrm{IPM}}$$

*"Effectively reduced cross sections: measured cross section scaled by that expected for a single-particle state with same energy and quantum numbers."*

# Macfarlane and French Sum Rules

Number of vacancies = $\sum (2j+1) SF_{\text{adding}}$

Number of occupancies = $\sum SF_{\text{removing}}$

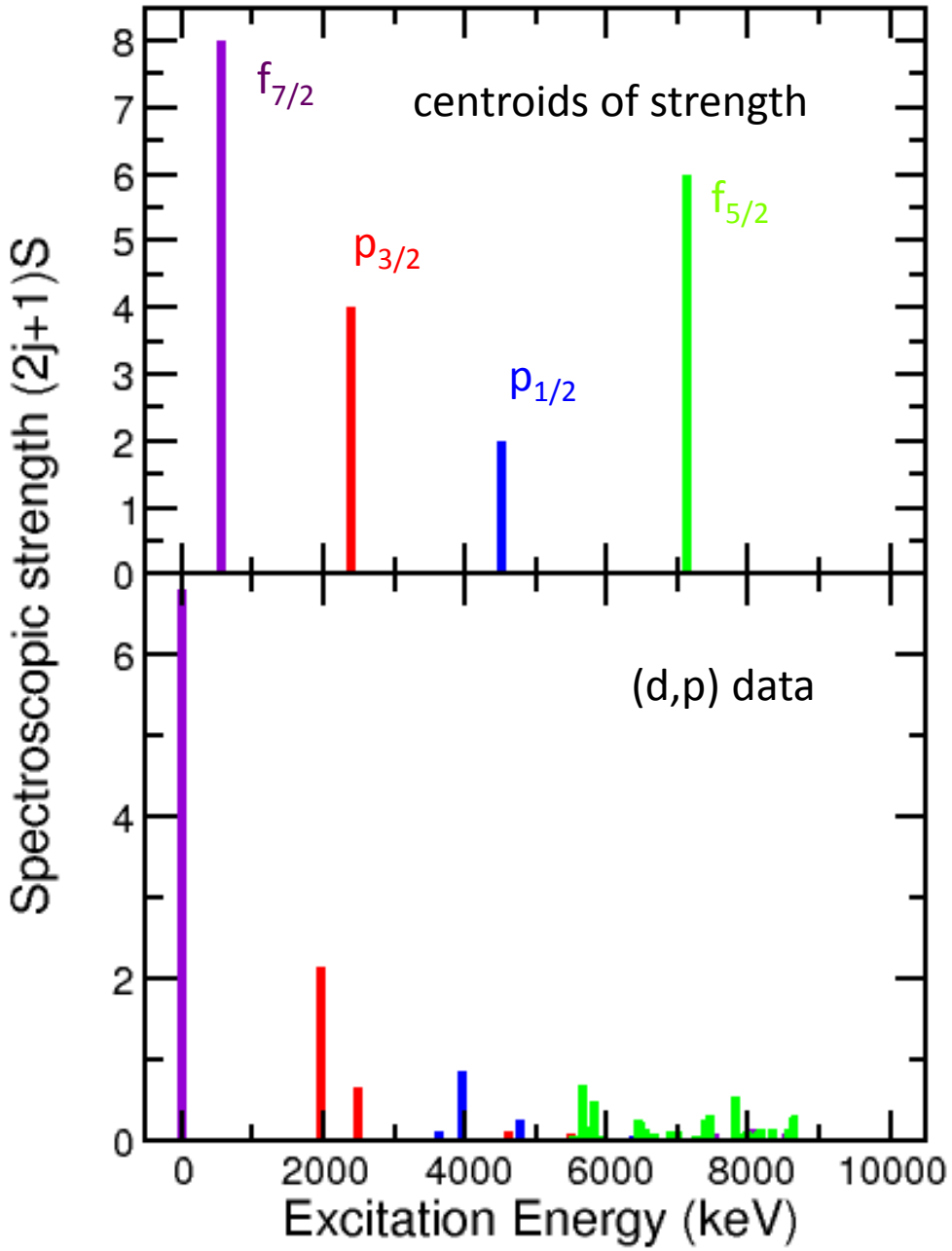
*Sums over all states populated via transfer of nucleon from the relevant orbit.*

Doing both adding AND removal reactions on the same target provides a quantitative check:

$$2j+1 = \sum SF_{\text{removing}} + \sum (2j+1) SF_{\text{adding}}$$

Adding all the occupancies in valence orbitals should yield the total number of valence nucleons — also provides a check.

Example: $^{40}Ca(d,p)^{41}Ca$ (neutron-adding)

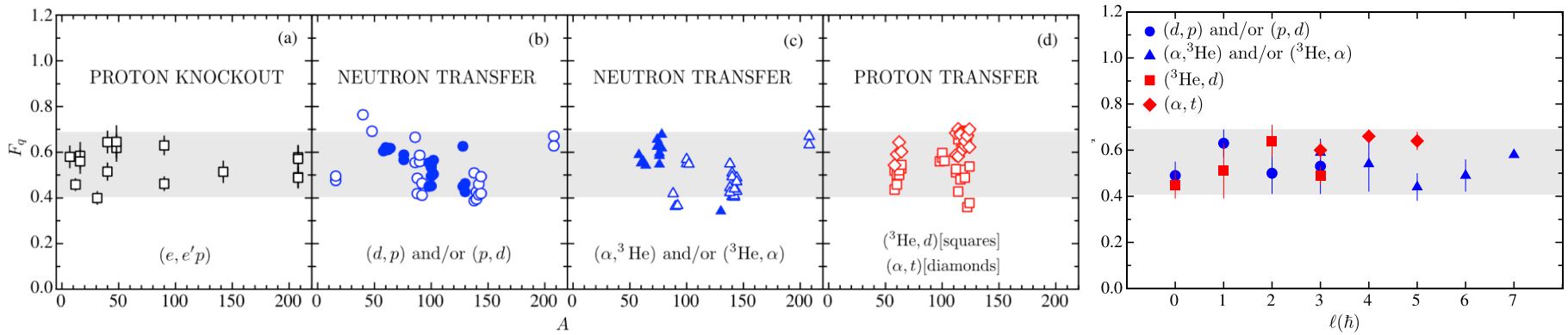


# Short Aside: Quenching of Cross Sections

Using these checks, find that cross sections appear quenched from expected single-particle values.

Cross sections appear quenched by 50-60% if consistent reaction modelling is used.

Analysis of 124 cases between $^{16}$O and $^{208}$Pb, induced by variety of reactions and $\ell$: quenching of 0.55.




Kay, Schiffer, Freeman PRL 042502 (2013)

Confirm/extends an effect identified in (e,e'p) studies in 1990's:

- Independent of whether nucleon added or removed, type of nucleon transferred, mass, reaction or angular momentum transfer.
- Appears to be a uniform property in large part thought to be due to the effects of short-range correlations (SRC).

In what follows, *choose to normalise* the observed population of a valence orbital to a maximum of *(2j+1)* using individually deduced quenching factors:

- Somewhat a convention; but if a uniform property this should not matter, makes them *relative* numbers.
- Indicates the reaction modeling and normalization adopted has a wider global consistency.

# Trivialisation of Experimental Methods

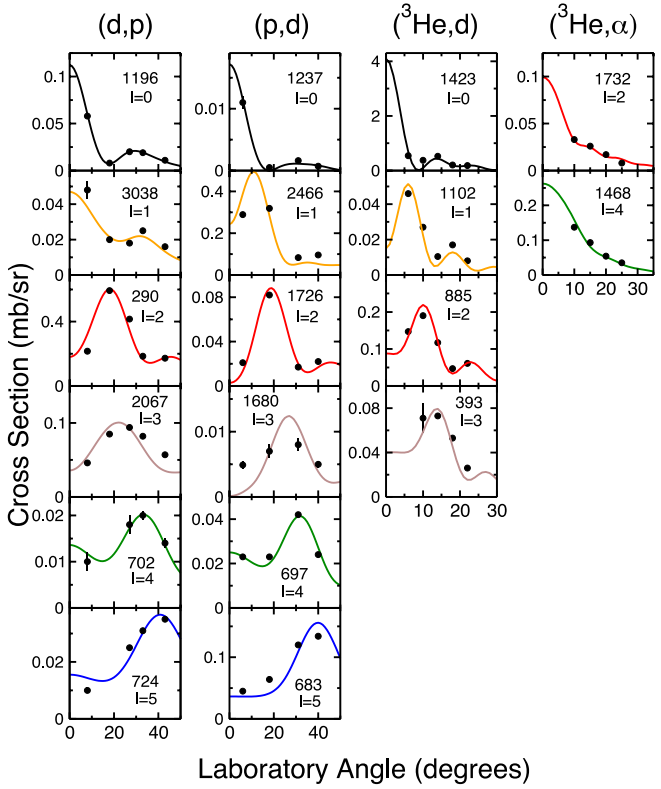
Light-ion induced reactions under conditions where a direct mechanism dominates.
*[10-20 MeV/u, forward angles, 1st peak]*

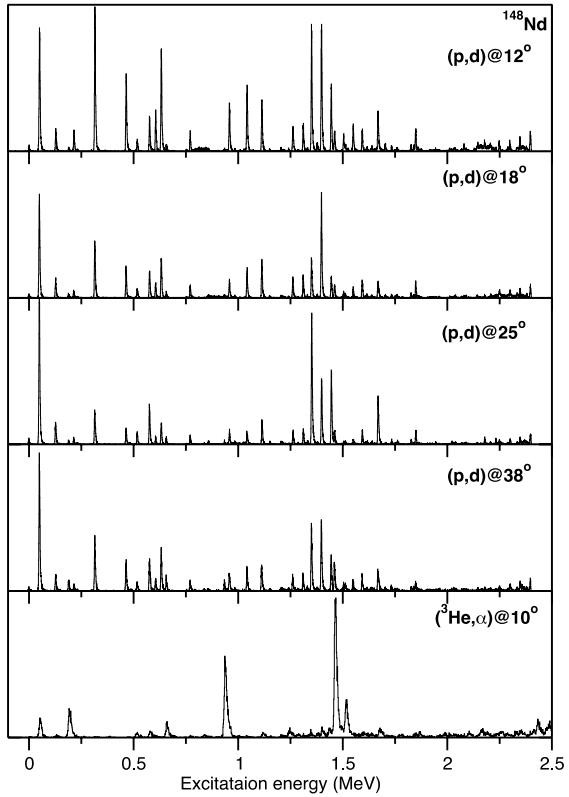
Two sets of reactions sometimes needed to meet "momentum-matching conditions" for good yields of low and high $\ell$ transfer. [e.g. neutron transfer by both (d,p) and ($\alpha$,$^3$He)]

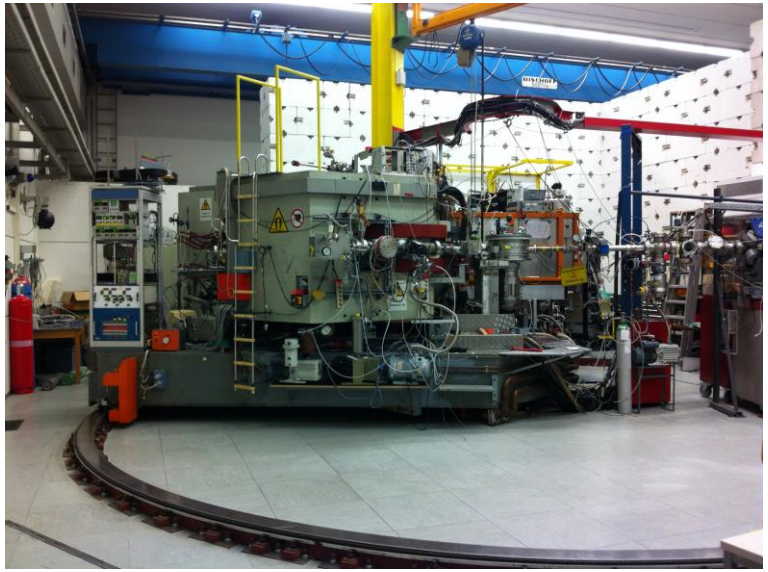
Identify outgoing ions on the basis of their momentum, dispersed using a magnetic spectrometer, and energy-loss characteristics in gas-filled focal plane detector.

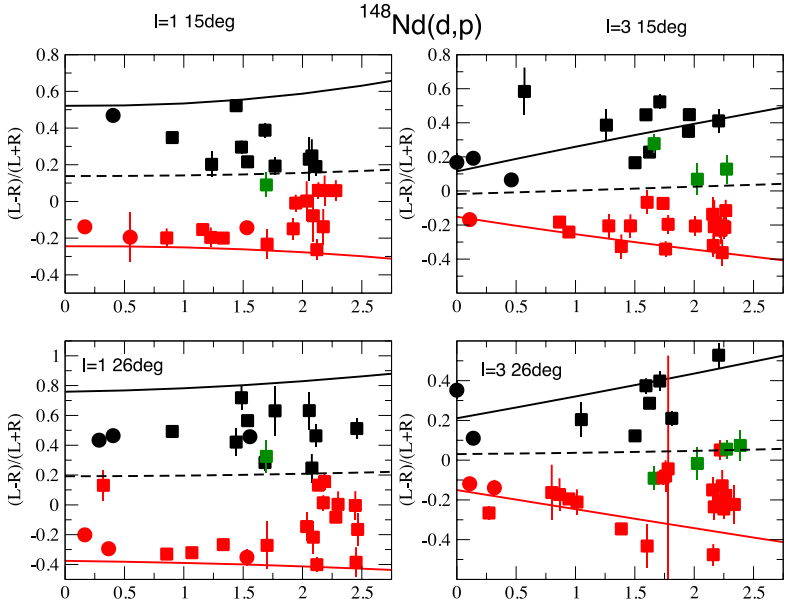
Measure cross sections to final states as a function of angle. [Absolute scale by comparison with elastic scattering in Rutherford regime.]
Measure asymmetries with polarised beams.
Deduce spin-parities of final states; although many already known.

*Be careful to ensure <u>consistency</u> in both the experimental methods and subsequent reaction modeling.*



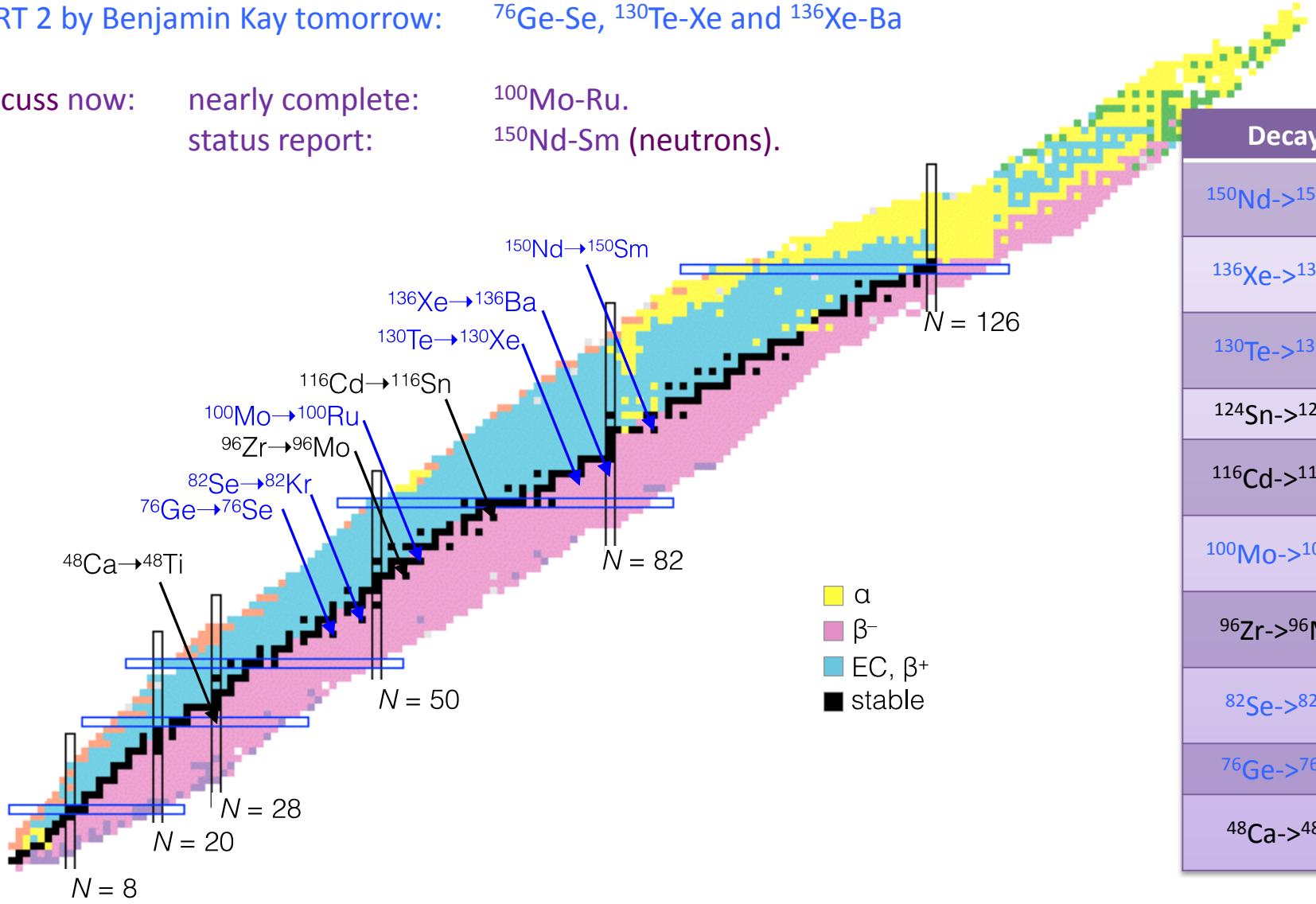


# Status of Our Programme of Measurements

Measurements on 0νββ candidates and daughters, and some neighbouring nuclei for consistency purposes.

PART 2 by Benjamin Kay tomorrow: $^{76}$Ge-Se, $^{130}$Te-Xe and $^{136}$Xe-Ba

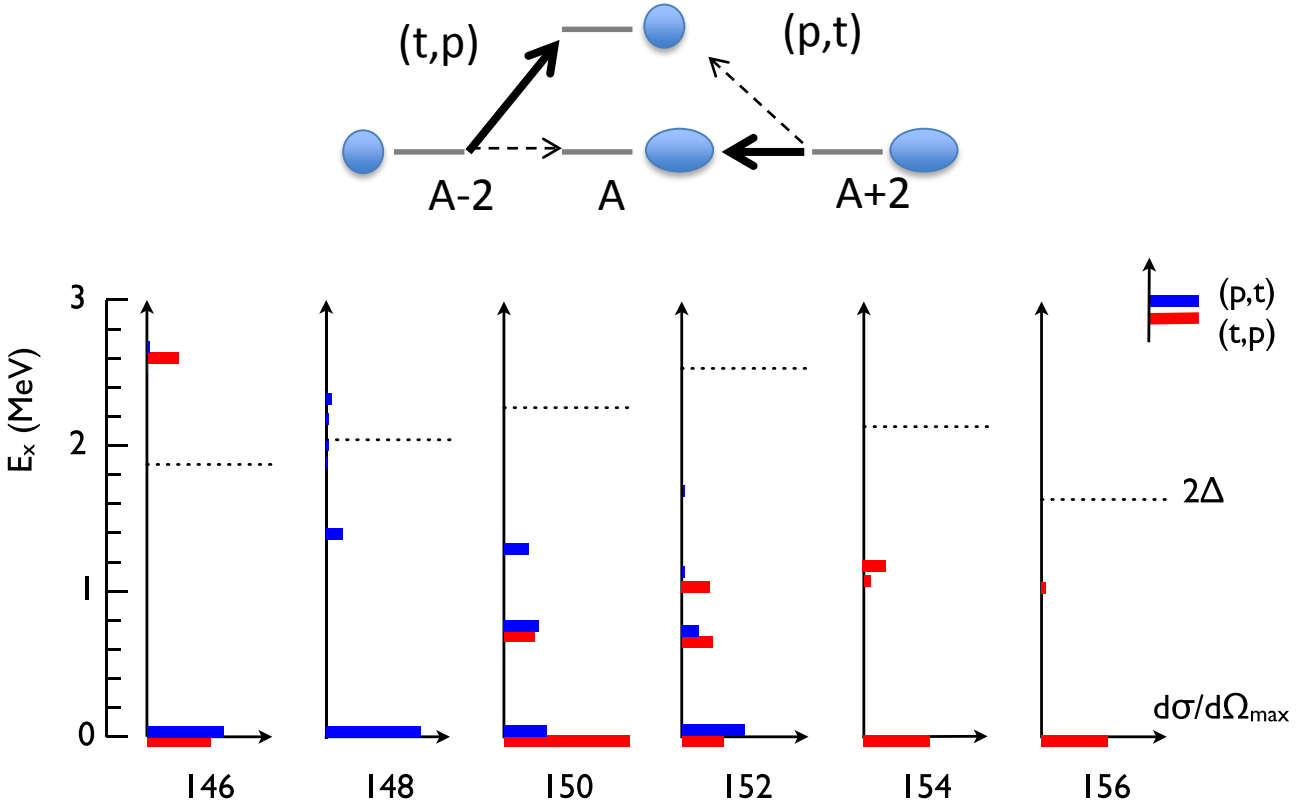
Discuss now: nearly complete: $^{100}$Mo-Ru.
status report: $^{150}$Nd-Sm (neutrons).




| Decay | Q (MeV) | $G_{0\nu}$ | % Abundance |
|---|---|---|---|
| $^{150}$Nd->$^{150}$Sm | 3.37 | 63.03 | 5.6 |
| $^{136}$Xe->$^{136}$Ba | 2.48 | 14.58 | 8.9 |
| $^{130}$Te->$^{130}$Xe | 2.53 | 14.22 | 34.5 |
| $^{124}$Sn->$^{124}$Te | 2.23 | 9.04 | 5.6 |
| $^{116}$Cd->$^{116}$Sn | 2.80 | 16.70 | 7.5 |
| $^{100}$Mo->$^{100}$Ru | 3.03 | 15.92 | 9.6 |
| $^{96}$Zr->$^{96}$Mo | 3.35 | 20.58 | 2.8 |
| $^{82}$Se->$^{82}$Kr | 2.99 | 10.16 | 9.2 |
| $^{76}$Ge->$^{76}$Se | 2.04 | 2.36 | 7.8 |
| $^{48}$Ca->$^{48}$Ti | 4.27 | 24.81 | 0.187 |

# A=150 Region

- N≈90 Sm nuclei are the classic example of shape transition effects – first seen in population of excited $0^+$ states in pair transfer; Nd nuclei show globally similar effects in (*p,t*) and (*t,p*).
- Distinct shape change between N=88 and 90 – lots of other evidence.




Samarium isotopes

(t, p) REACTION

on the basis of the present work alone
igned from the shapes of the (t, p) angu

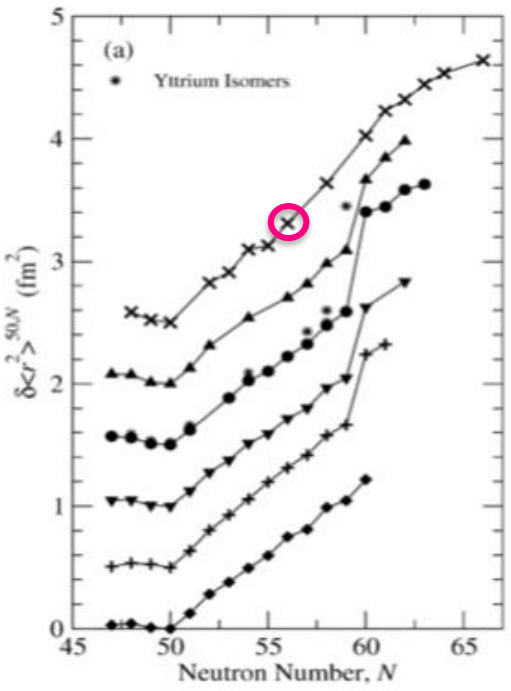
RUCTURE INFORMATION

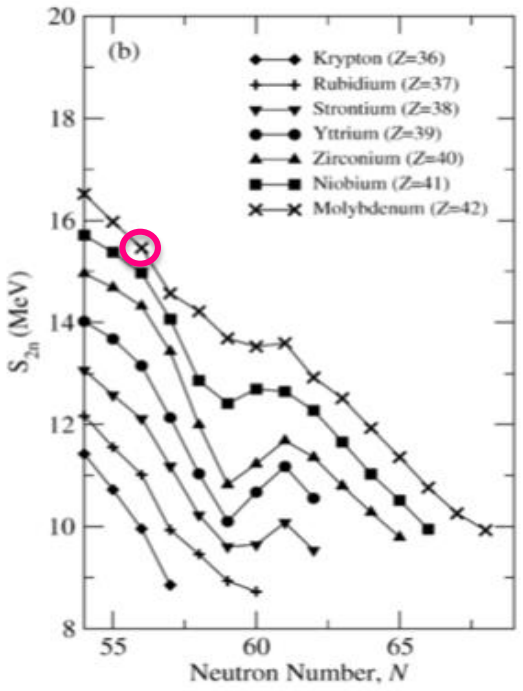
n for two-nucleon transfer reactions dep
amplitudes of the configurations whic
states excited [24]). For this reason, in
it is impossible to extract from the exper
vave function. Indeed, nuclear model wa
tative analysis of two-nucleon transfer r
ctions in the region presently under stud
qualitative one based on the general pr
ucleon transfer studies of heavy nuclei

*Bjerregaard NP 86, 145 (1966), Debenham NP A195, 385 (1972), Chapman NP A186, 603 (1972), Alford NP A321, 45 (1979).*

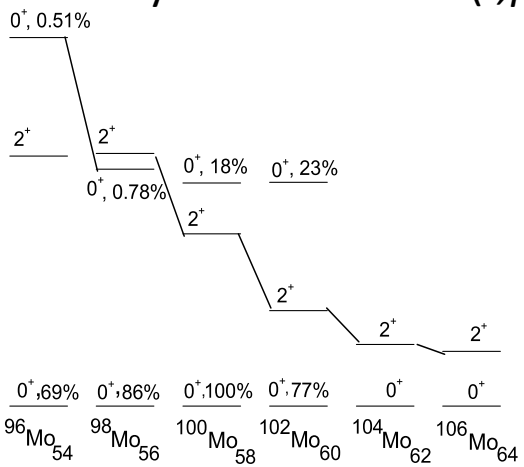
# A=100 Region

*Charlwood PLB 674(2009)23*



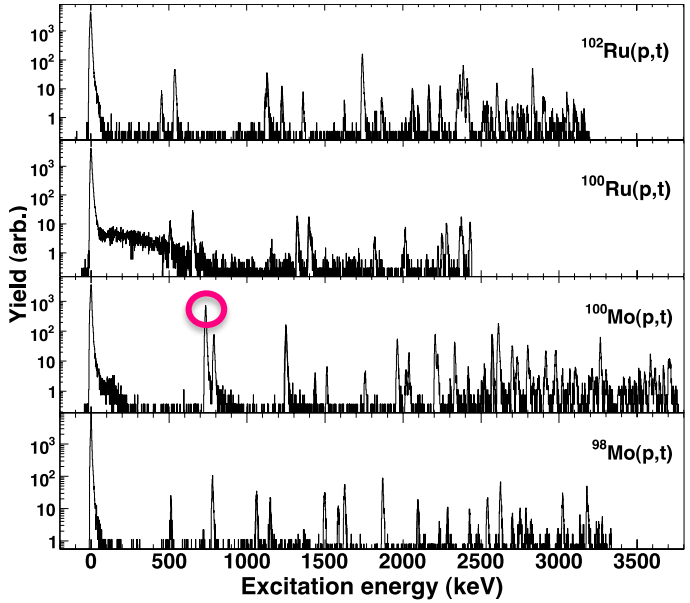

## Systematics from (*t,p*)




Transitions strengths normalised to $^{100}Mo_{gs}$.

*Rahman et al. PRC 73, 054311 (2012)*

## Spectra from (*p,t*)




*J.S. Thomas et al. PRC 86,047304(2012)*

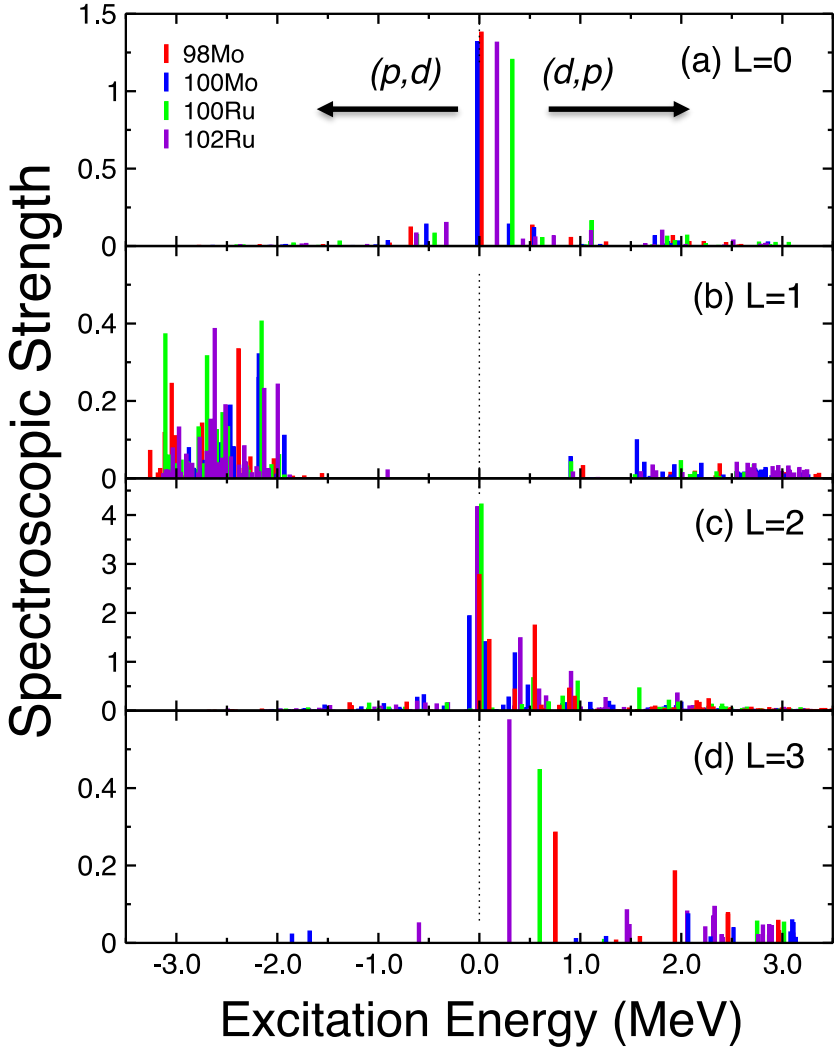
- Population of low-lying $0^+$ states in (t,p) and (p,t) to $^{98\text{-}102}$Mo and (t,p) to $^{104,106}$Ru.
- Mean-square charge radii show no sudden shape change for Mo or Ru – smooth transition.
- Some evidence of soft and triaxial behaviour in γ spectroscopy of Mo and Ru fission fragments.

A=100 nuclei are soft and transitional. A=150 more distinct transition from spherical to well deformed. Presence of deformation makes life <u>DIFFICULT</u> – presents experimental challenges in the large fragmentation of spherical single-particle strength. Presents theoretical challenges for calculation of ββ matrix elements where adifference in deformation of parent and daughter is likely to inhibit decays

# A=100 Experiments

Consistency checks – reactions on four targets.

Neutrons – low L – (d,p) and (p,d)




Neutron valence orbits:

$s_{1/2}$ $d_{3/2,5/2}$ $g_{7/2}$ $h_{11/2}$

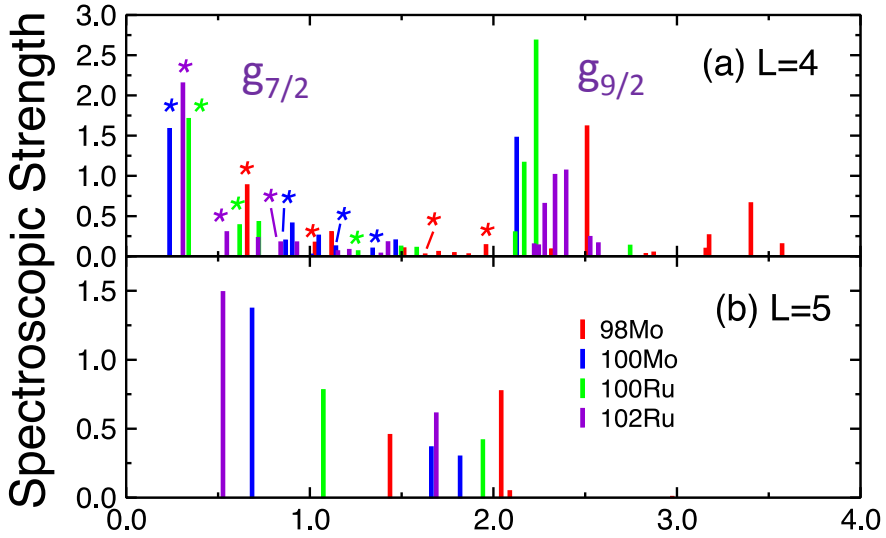
Proton valence orbits:

$p_{1/2,3/2}$ $f_{5/2}$ $g_{9/2}$

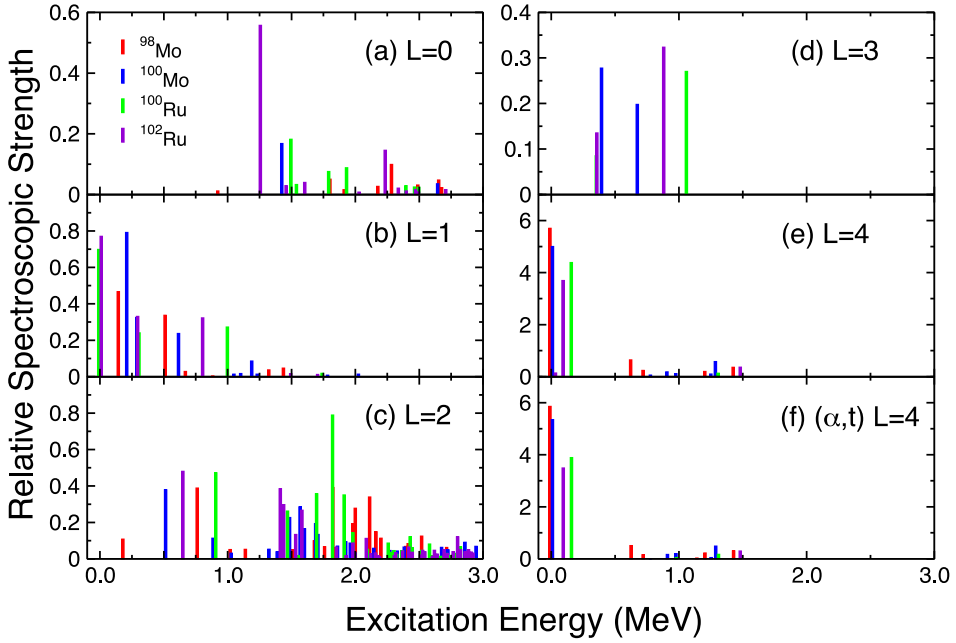
Estimates of "missing" strength for valence orbitals of the order of few %.

Maier-Leibnitz-Laboratorium
für Kern-, Teilchen- & Beschleunigerphysik

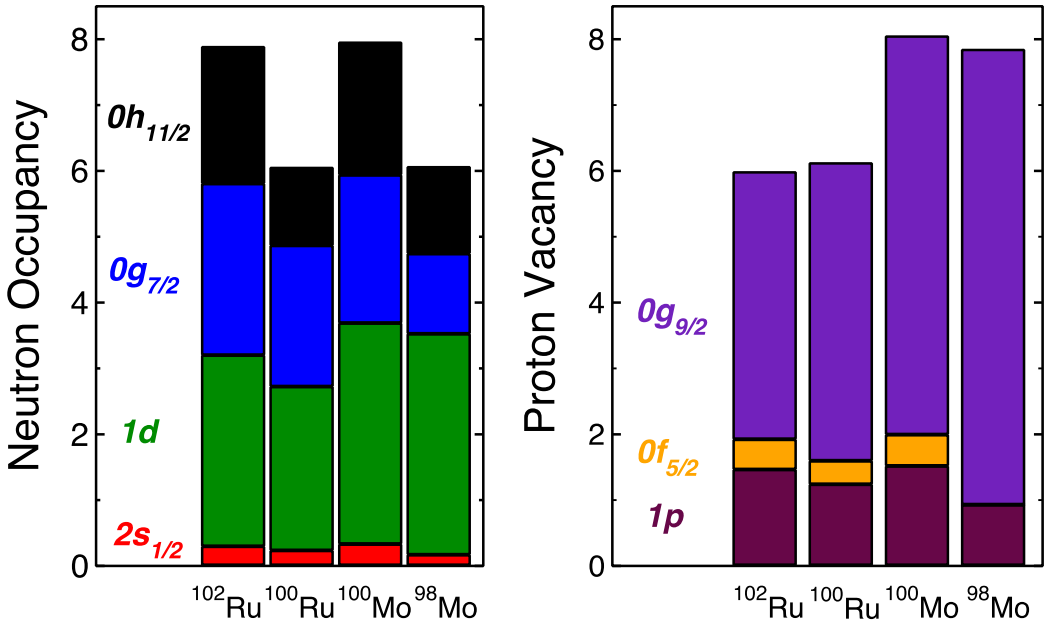
Neutrons – high L – (h,α)




Protons – all L – (h,d)



# A=100 Experiments – Reaction Modelling




Use Macfarlane-French sum rules to determine the measured strength and deduce "quenching factors" by comparison with appropriate quantities e.g. orbital degeneracy or number of valence nucleons.

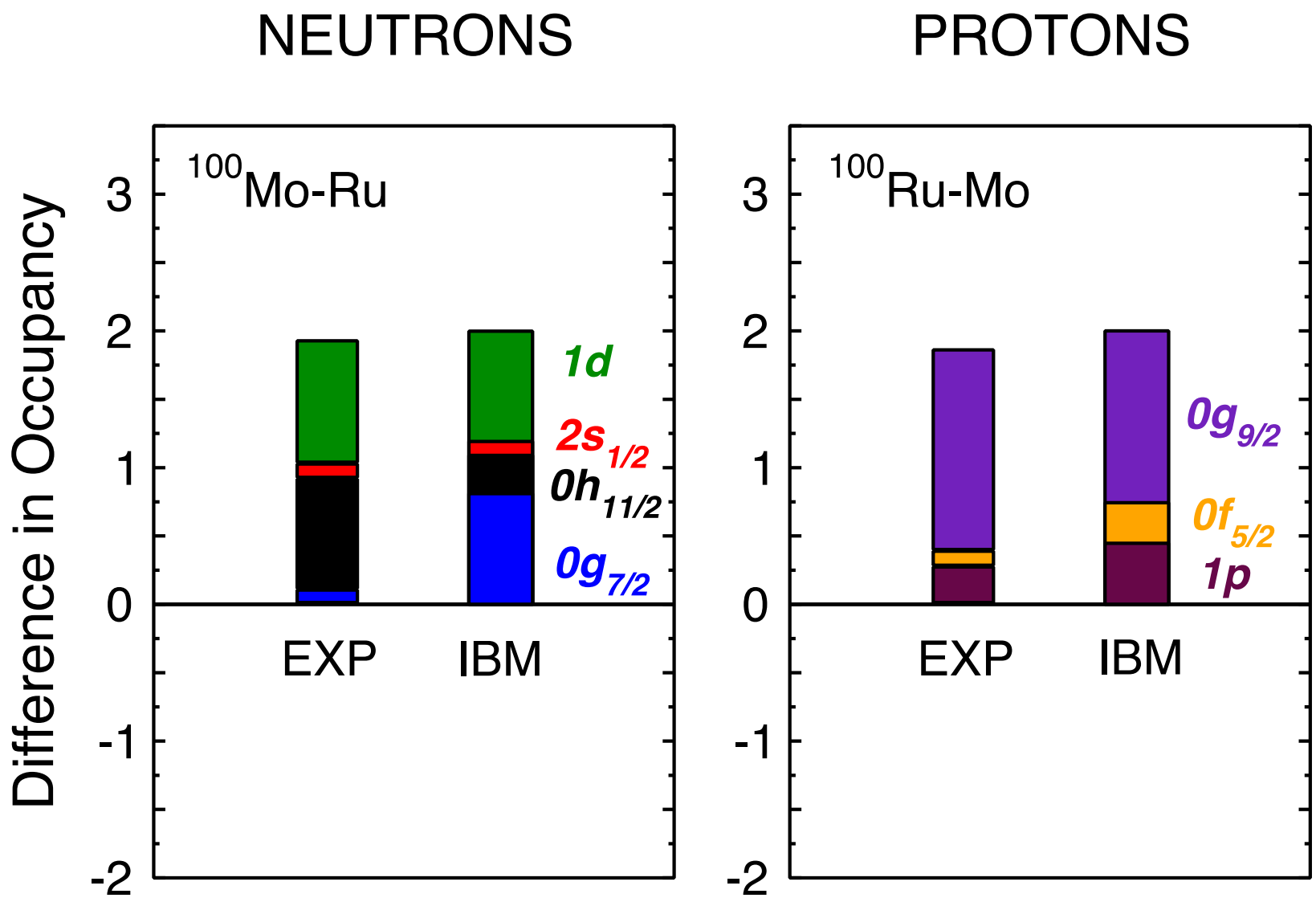
| | | | |
|---|---|---|---|
| (dp+pd): quenching factor of | 61.7% with only a | 5.6% rms deviation across four targets | for neutron L=0 and 2. |
| (h,a): | 55.0% | 6.1% | for neutron L=4 and 5. |
| (h,d): | 65.9% | 7.3% | for proton L=1, 3 and 4. |

Internal consistency across targets, between reactions and with previous "global studies" gives faith that no issues are creeping into the analysis.

Deduce the neutron occupancies from neutron-removal reactions and proton vacancies form proton-adding reaction – moving towards the closed shell at nucleon number 50, keeps the relevant strength low in energy and minimises risk of missing it!

Need to complete a detailed analysis of uncertainties – but relative occupancies can usually be determined to within a few tenths of a nucleon.

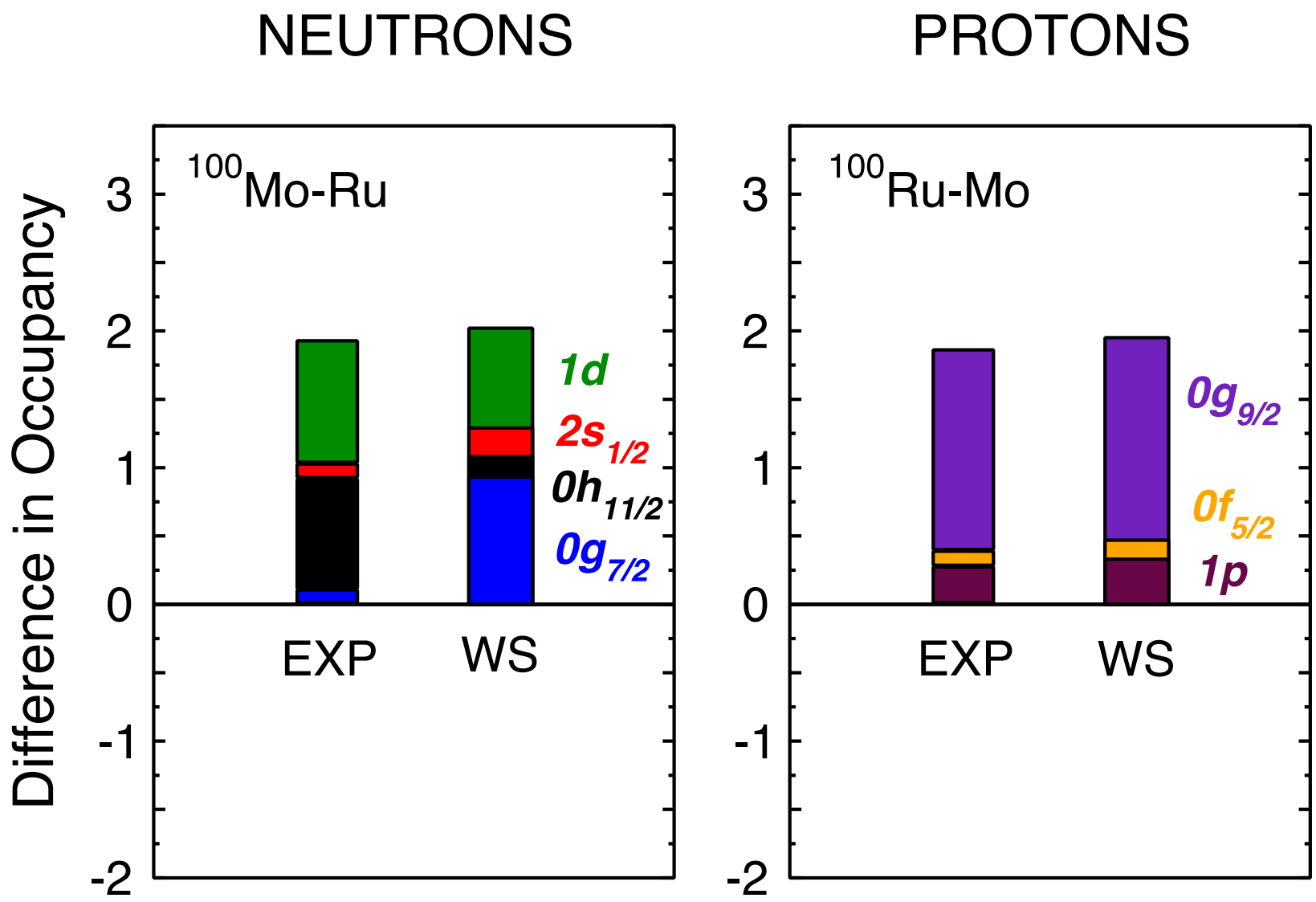
A=100




IBM: Seems to do quite well with protons.
Underestimates $\mathbf{h_{11/2}}$ and overestimates $\mathbf{g_{7/2}}$

*Kotila and Barea PRC 94,034320 (2016)*

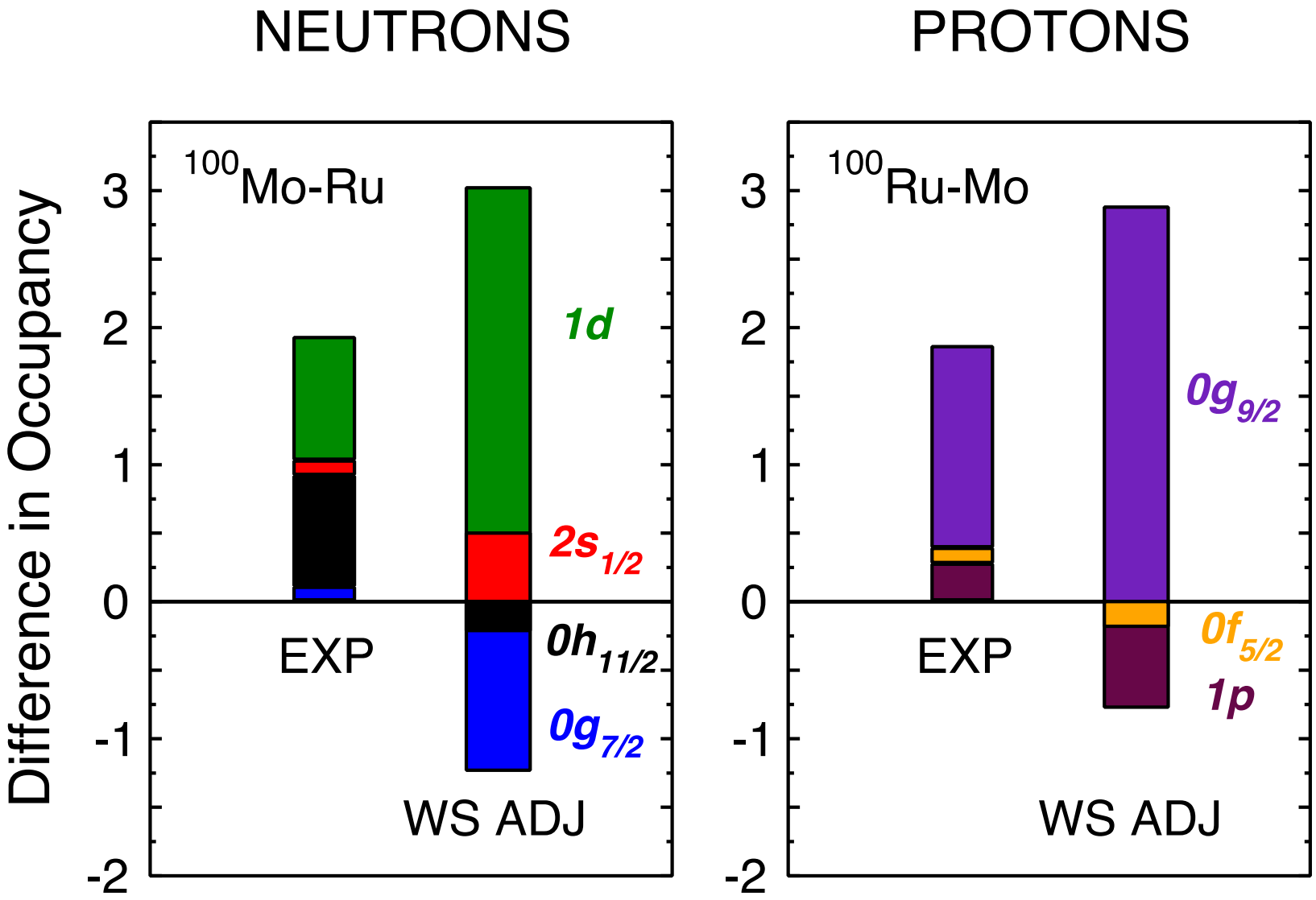
A=100




WS+BCS: Seems to do quite well with protons.
Underestimates $\mathbf{h_{11/2}}$ and overestimates $\mathbf{g_{7/2}}$

*Suhonen, Private Communication*

A=100



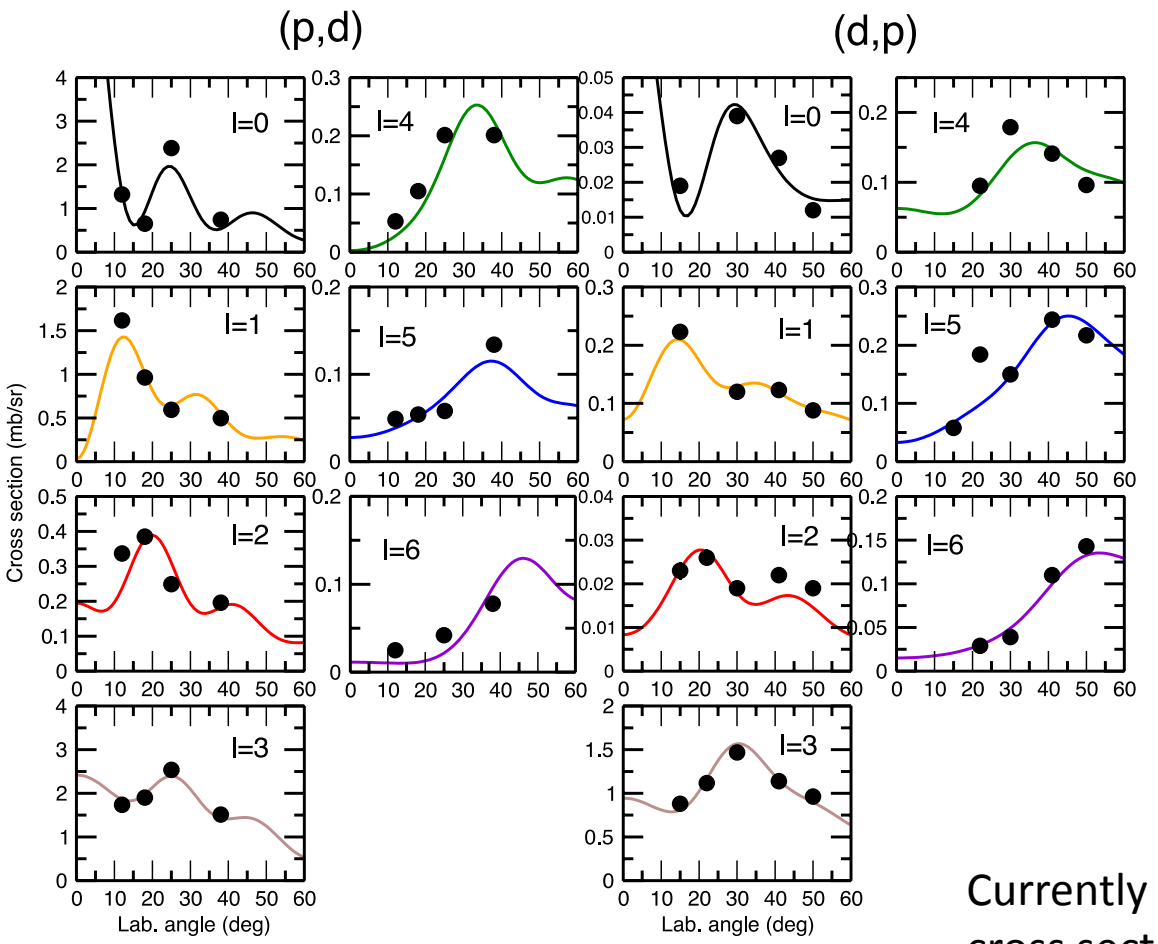

"Small modifications of the WS energies were done for $^{100}$Mo, $^{100}$Ru, $^{128}$Te and $^{128}$Xe at the vicinity of the proton and neutron Fermi surfaces to allow for a better reproduction of the one-quasiparticle type of spectra of the neighboring odd-A nuclei. ''

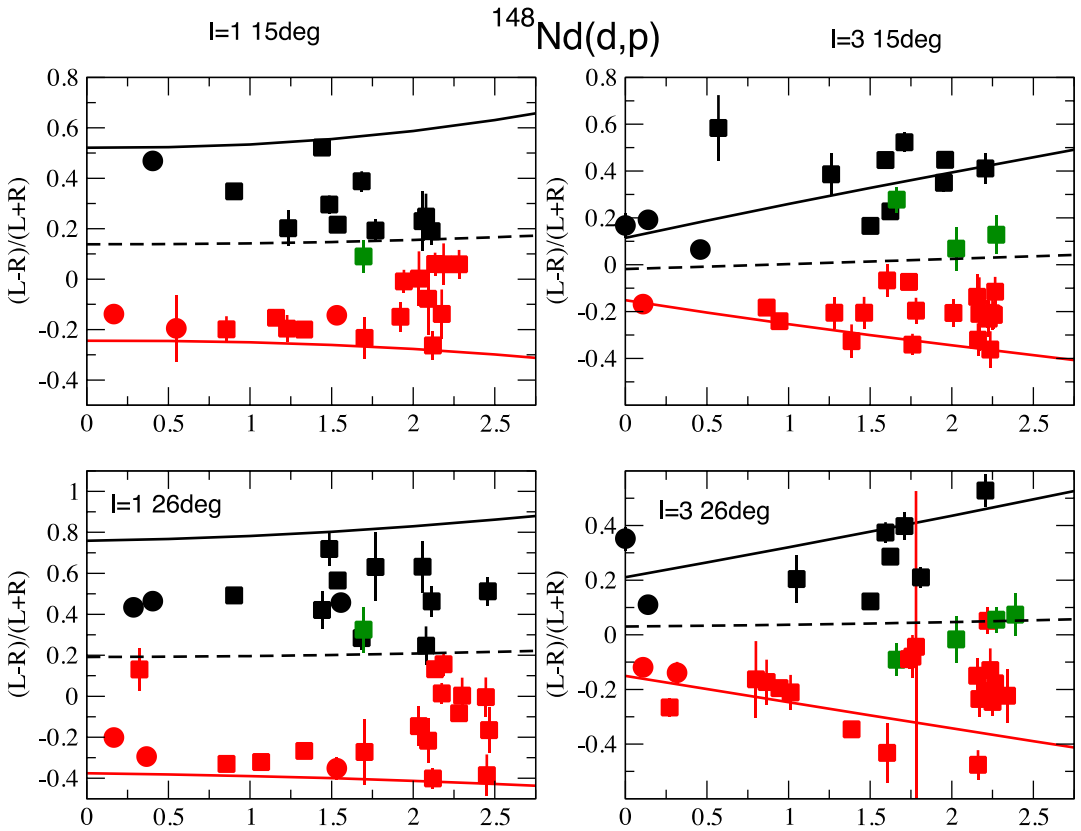
ADJ WS+BCS: Has the importance of losing **d** neutrons and gaining **g$_{9/2}$** protons – but over estimates the scale of the changes.

*Suhonen and Civitarese, NPA 924 (2014)*

# A=150 Experiments

(d,p) and (p,d) reactions on $^{148,150}$Nd and $^{150,152,154}$Sm – for consistency and to span the deformation changes.
Polarised and unpolarised proton and deuteron beams to assist assignment of j.
Considerable amount of analysis given the level densities in these systems.



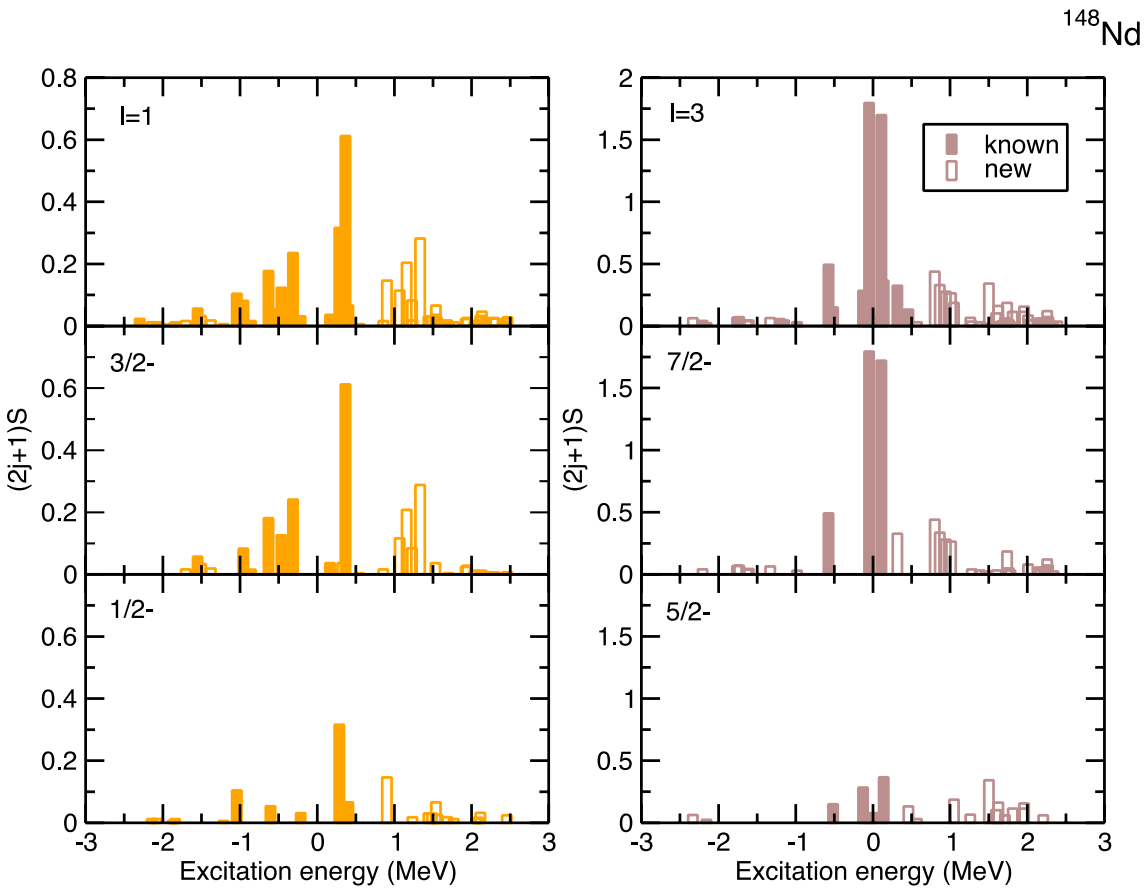



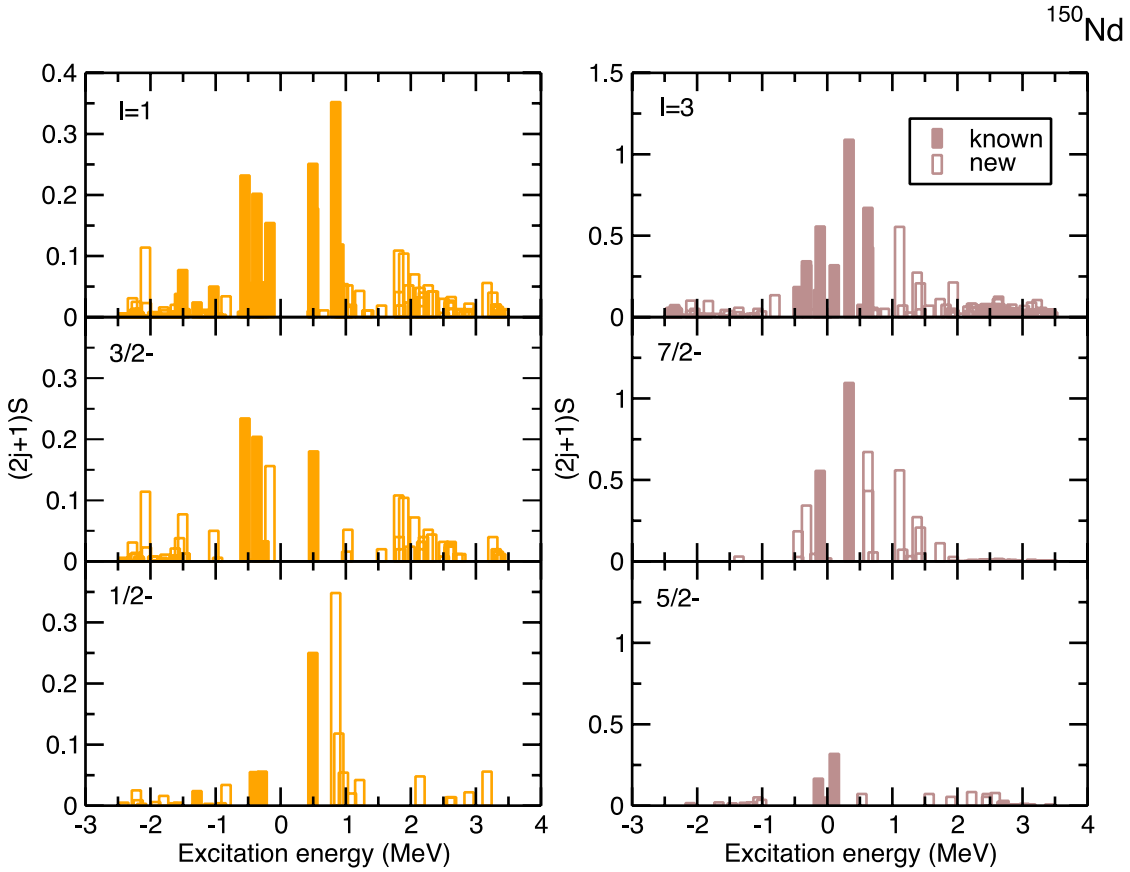

Currently completed all assignments to final states and have extracted cross sections.

# A=150 Experiments

Much wider spread of strength than in A=100.

Quenching factor for $f_{7/2}$ states which close to the Fermi surface is similar to global studies.



# Conclusions and Outlook

- Work on $^{100}$Mo will be published soon – some more work to do on $^{150}$Nd.

- $M^{0\nu}$ from different theoretical methods compare better than perhaps they did. Does the comparability of different theoretical measurements actually guarantee that they are correct? Is it prudent require benchmarks against other *relevant* nuclear properties?

- Useful checks appear possible by comparison with measurements of specific nuclear properties such as the changing occupancies of valence nucleon orbits and pair transfer studies (not discussed here). Ben will mention impact of $^{76}$Ge measurements – it would be useful to make similar theoretical consideration of our data on $^{100}$Mo-Ru, $^{130}$Te-Xe and $^{136}$Xe-Ba.

- What other nuclear properties might have a critical connection with $M^{0\nu}$ and be used as constraints on calculations? This question has been asked frequently enough…but are we clear about the answer?

- Theoretical calculations are not always hygienic enough to disentangle the physics that matters for $M^{0\nu}$ into a simple "intuitive" understanding – at least from an experimentalist point of view – for such a rare process, does even one exist?

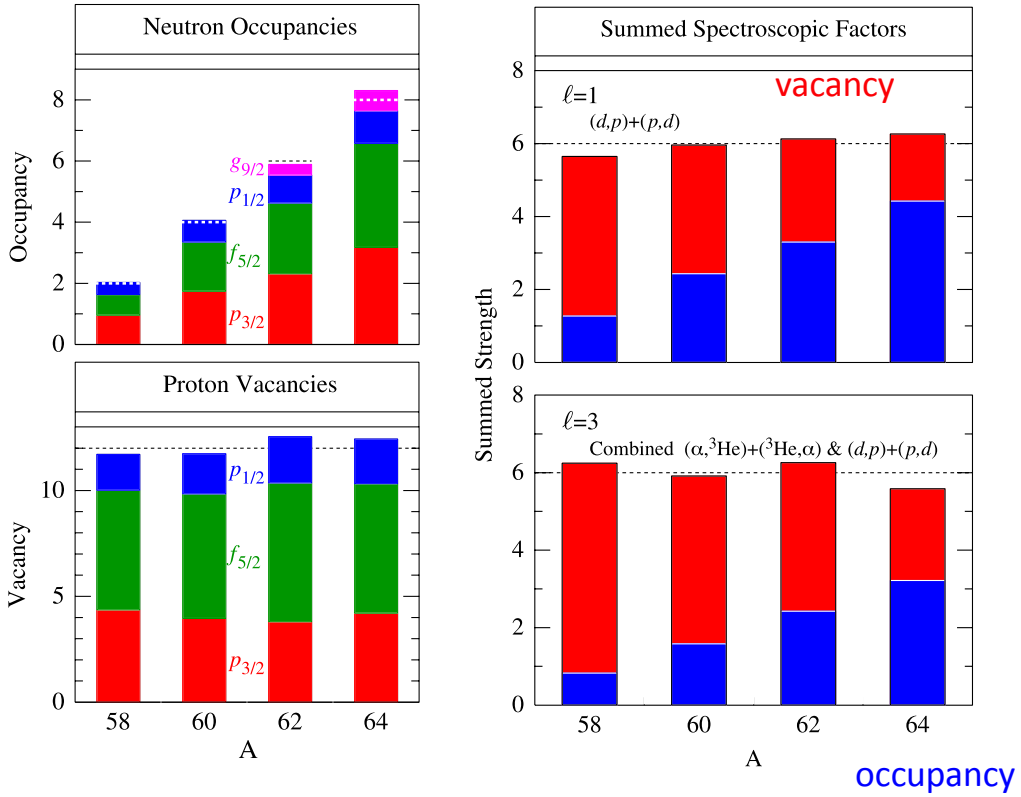
THE END

# Are SF observables?

In the strictness sense “NO”… they are dependent on reaction model used.

But do seem to provide a self-consistent picture of occupancies, at least when the same consistent normalisation procedures are adopted.

- Summed occupancies behave in a consistent way over ranges of isotopes in cases where the number of valence nucleons changes or remains the same. And this happens for different mass regions.
- Where both are measured, the relative number of holes and particles change in a way consistent with the expected populations.



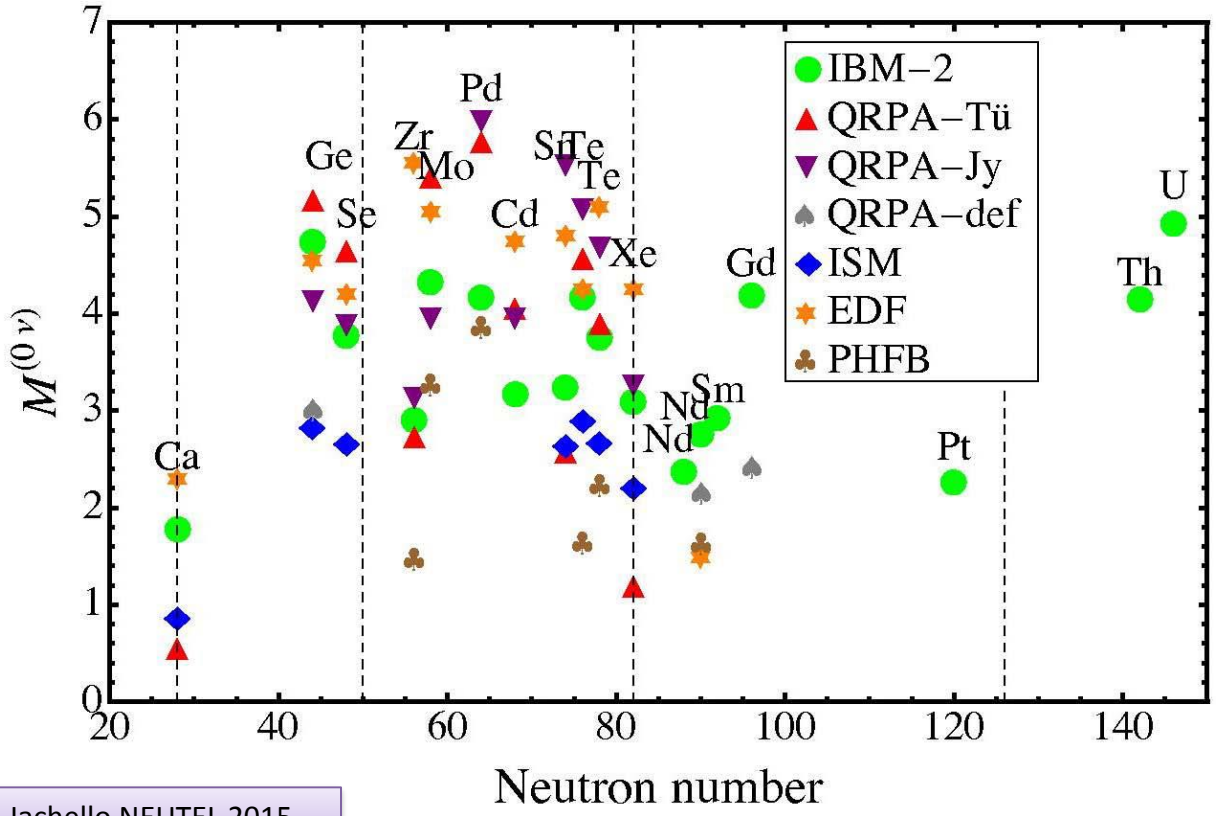

From transfer data on four stable Ni isotopes

# Neutrino Masses

For neutrinoless double beta decay mediated by a light massive neutrino:

$$\text{Rate} = G^{0\nu} \, |M^{0\nu}_{\text{GT}} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_{\text{F}}|^2 \, \langle m_\nu \rangle^2$$

Convincing observation: Majorana neutrinos and but their absolute mass scale?




NO other experimentally accessible simple process can directly determine the nuclear matrix element: HAVE to rely on calculation.

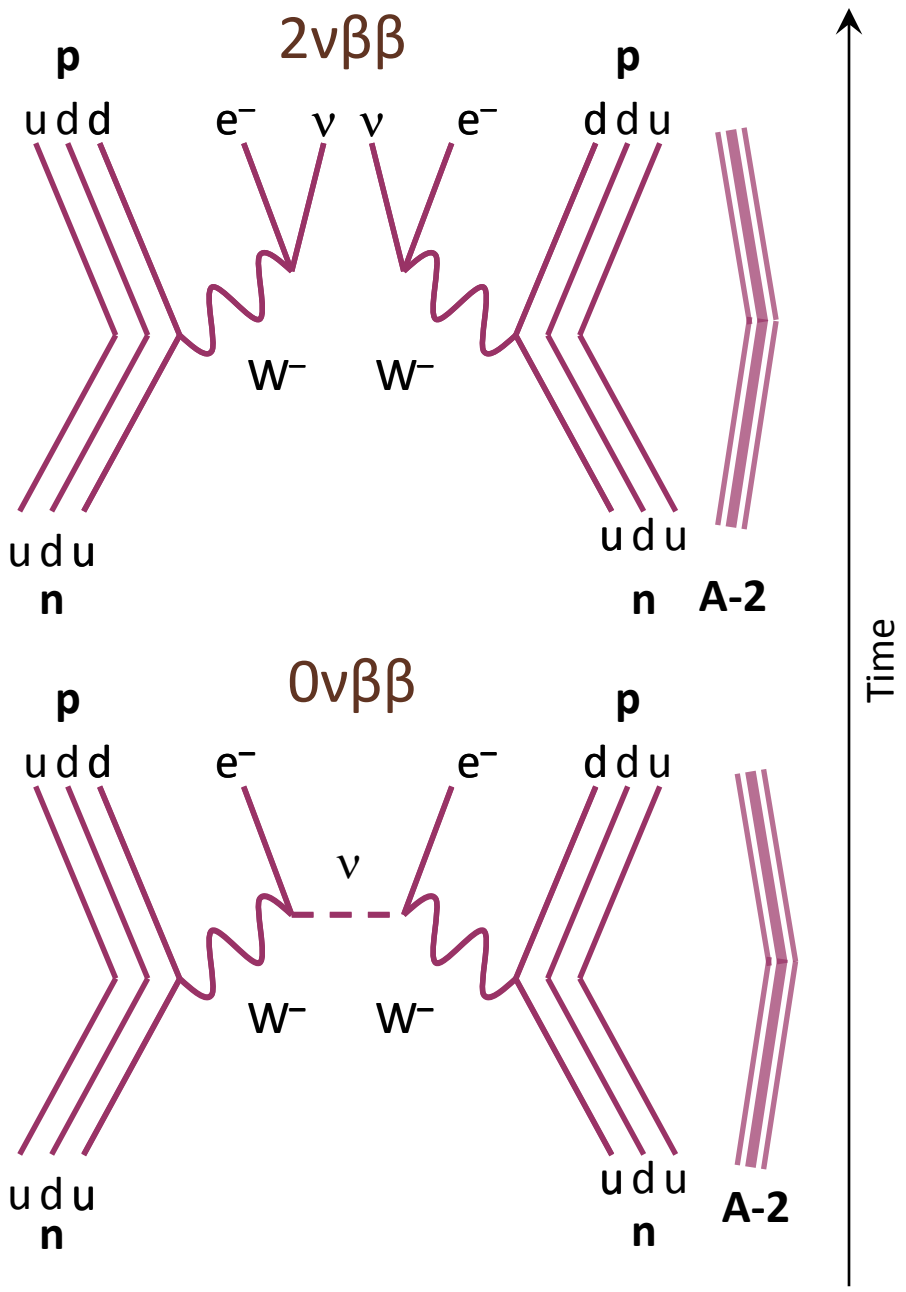
Iachello NEUTEL 2015.

Double beta decay with neutrinos:

- simultaneous ordinary beta decay.
- lepton number conserved and SM allowed.
- observed in around 10 nuclei with $T_{1/2} \approx 10^{19-21}$ years.

Double beta decay without neutrinos:

- lepton number violated and SM forbidden.
- no observations, $T_{1/2}$ limits are $10^{24-25}$ years.
- simplest mechanism: exchange of a light massive Majorana neutrino
  i.e. neutrino and antineutrino are identical.
- Other proposed mechanisms, but all imply Majorana neutrinos with mass (Schechter-Valle theorem).



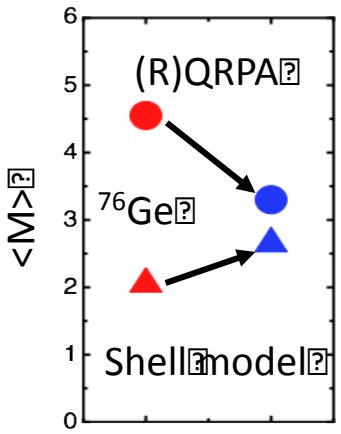
# Valence Nucleon Occupancies Relevant to 76Ge

Neutron and proton adding and removal reactions on $^{76,74}$Ge and $^{76,78}$Se targets.

Both proton and neutron Fermi surfaces in shell that includes: $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$ and $0g_{9/2}$.

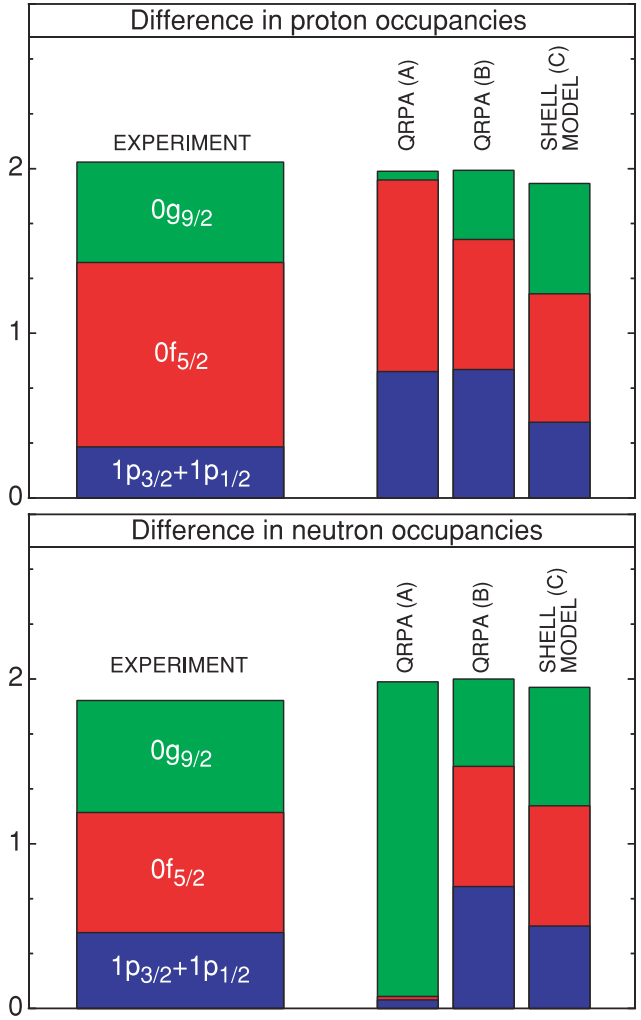
Experimentally deduced changes in a putative 0νββ decay compared to prior theoretical calculations (A) suggest:

- $0g_{9/2}$ protons considerably more involved.
- $0g_{9/2}$ neutrons considerably less involved.
- Both Fermi surfaces more diffuse.




Calculations with adjusted mean fields (B) and (C) :

- In QRPA, $M^{0\nu}$ fell by around 30%.
- In SM, $M^{0\nu}$ increased by 15%.
- Discrepancy reduced by factor two.




PRL 100 112501 and PRC 79 021301(R)

Facilities used at:

Yale University

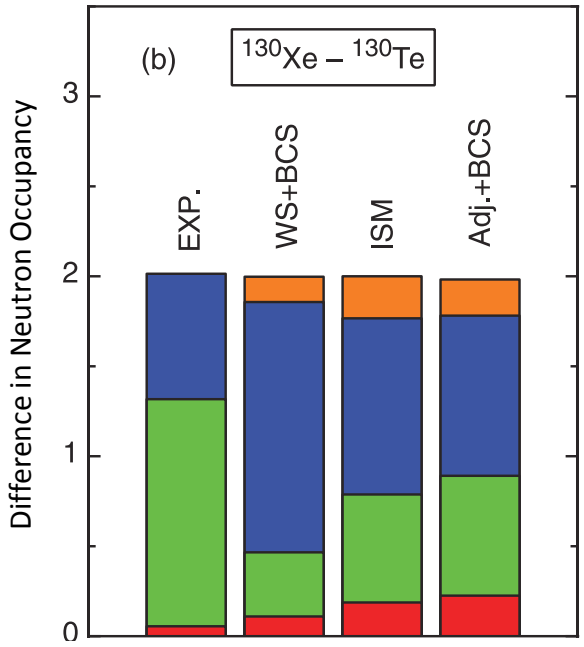
# Valence Neutron Occupancies Relevant to 130Te

**Neutron** adding and removal reactions on $^{128,130}$Te and frozen $^{130,132}$Xe targets.

Neutron Fermi surfaces in shell that includes: $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ and $0h_{11/2}$.

Experimentally deduced changes in a putative 0νββ decay compared to prior theoretical calculations suggest:

- $0g_{7/2}$ fully occupied and inactive; no evidence for population at low excitation energy.
- Relative roles of other orbitals differ from those in the calculations of 0νββ matrix elements.




PRC 87 011302(R)

Facilities used at:

Yale University

**Protons**:

- Old data for Te targets only suggest no proton $0h_{11/2}$ strength despite playing a role in calculations of 0νββ. Consequence of a Z=64 subshell gap?
- Recent experiment: data from proton transfer on solid Te and Ba targets and Xe gas cells under analysis.

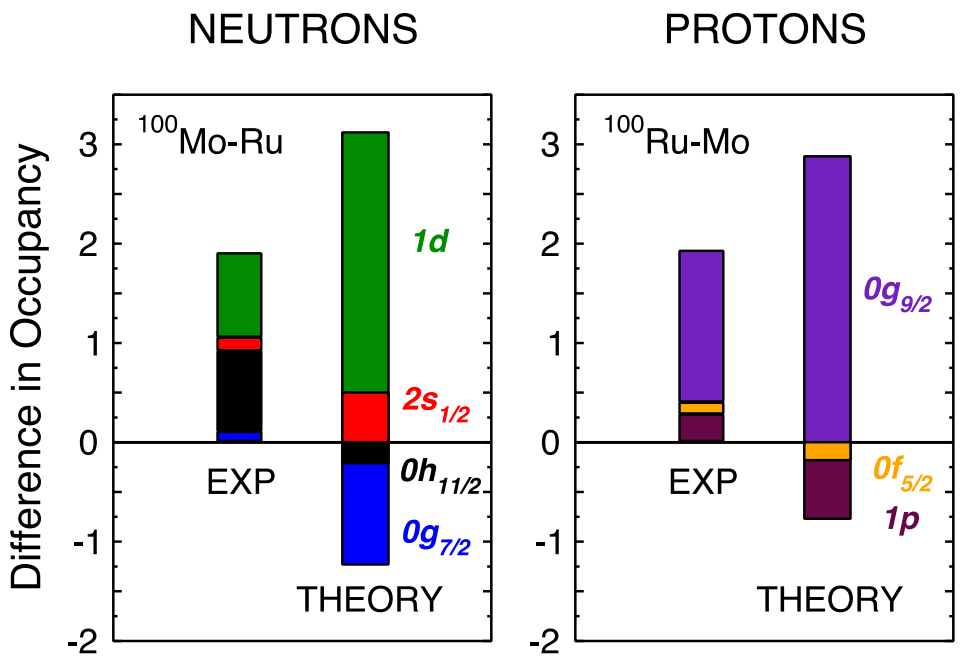
# Valence Nucleon Occupancies Relevant to 100Mo

Neutron and proton adding and removal reactions on $^{98,100}$Mo and $^{100,102}$Ru targets.

Proton Fermi surfaces in shell that includes: $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$ and $0g_{9/2}$.

Neutron Fermi surfaces in shell that includes: $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ and $0h_{11/2}$.

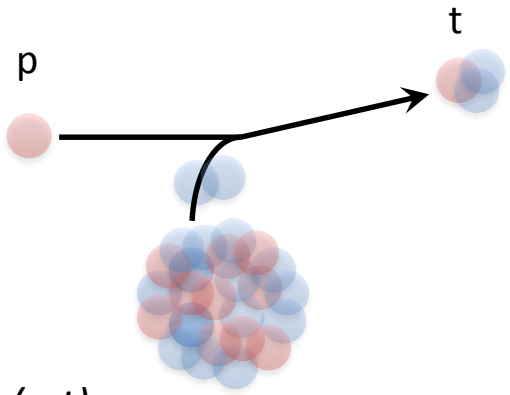
Recently published QRPA occupancies are different; some neutron orbitals *increasing* and some proton orbitals *decreasing*.




Publication in preparation

Facilities used at:

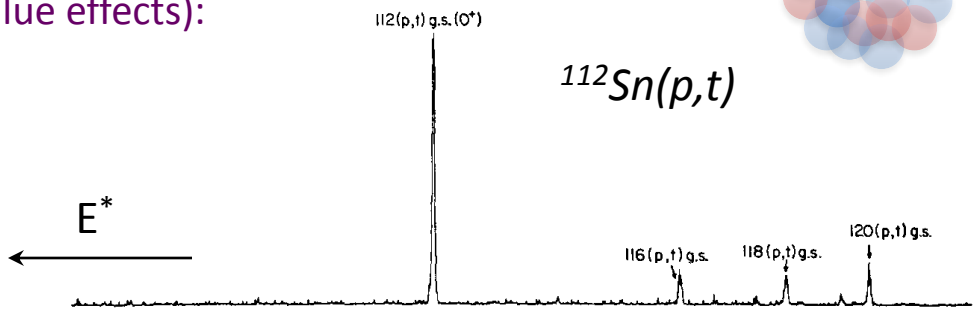
# Pairing and Two-Nucleon Transfer




BCS pairing assumed in QRPA approaches, which can be tested in pair transfer reactions [(t,p),(p,t) and ($^3$He,n)] where correlations enhance cross sections to "well-paired" 0+ states.

Simple estimate (ignoring Q value effects):

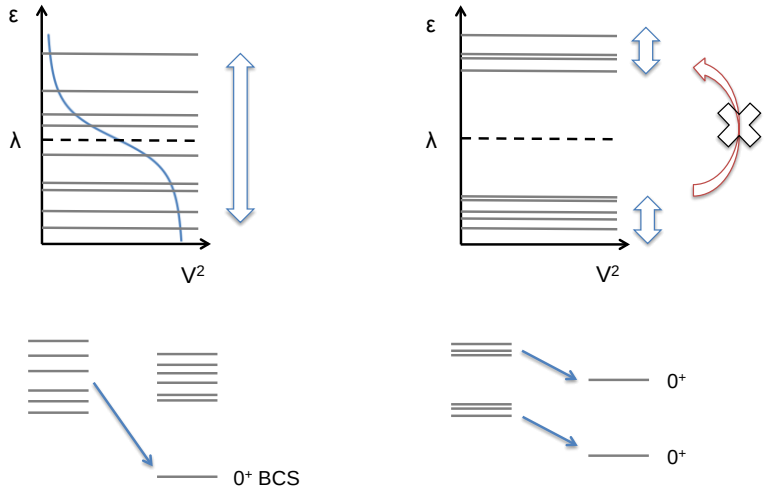
$$\frac{\sigma_{\mathrm{gs}\to\mathrm{gs}}}{\sigma_{\mathrm{gs}\to 2\mathrm{qp}}} = \left[\frac{\Delta}{GU_\nu^2}\right]^2 \sim \frac{A}{4}$$



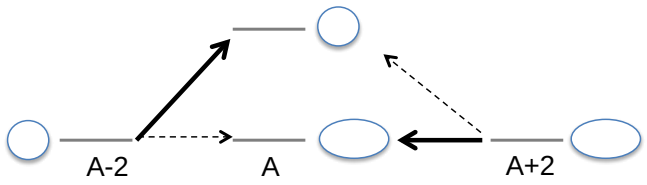

*Fleming et al. NP A281, 389 (1977)*

Picture breaks down with strongly populated $0^+$ excited states, when:

(i) Failure of BCS due to subshell gaps



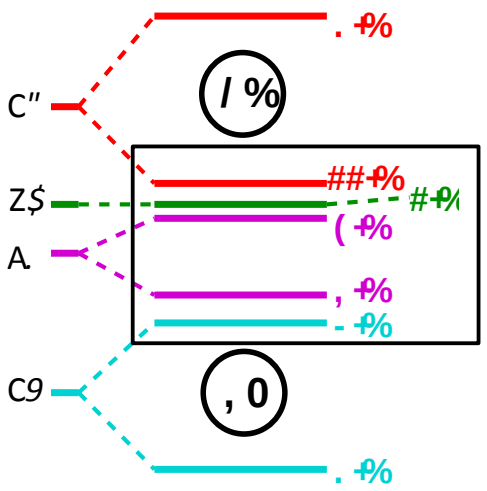

(ii) Regions of shape transitions

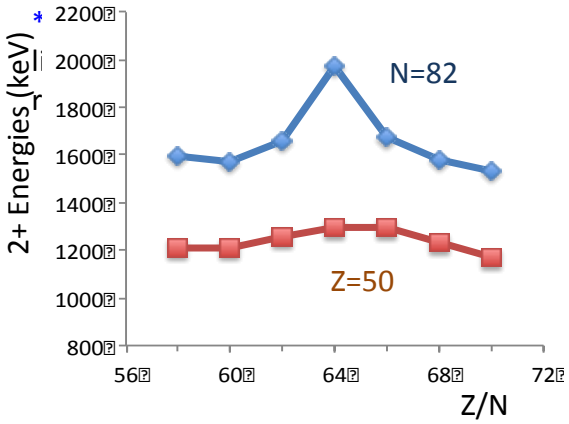



The population of excited $0^+$ states is often referred to as the presence of pairing vibrations.

# Two-Nucleon Transfer Results

| SYSTEM | KEY DATA* | FINDINGS |
|---|---|---|
| $^{76}$Ge-Se | *(p,t)* PRC 75,051301 (2007)<br>*(3He,n)* PRC 87,051305 (2013) | Excited 0+ states at the level of few % of gs strength. GS reduced cross sections very similar. |
| $^{130}$Te-Xe | *(p,t)* PRC 82,027308 (2007) & PRC 87,011302 (2013)<br>*(3He,n)* NP 323, 339 (1979) | As above for neutrons.<br>Proton-pairing vibrations due to Z=64 subshell gap. |
| $^{100}$Mo-Ru | *(p,t)* PRC 86, 047304 (2012)<br>*(t,p)* PRC 73, 054311 (2012) & NP A184, 357 (1972)<br>*(3He,n)* NP A269, 125 (1976) | Neutron-pairing vibrations due to shape coexisting states. Transitional region with onset later i |
| $^{136}$Xe-Ba | *(p,t)* NP A341, 206 (1980)<br>*(3He,n)* NP A269, 125 (1976) | Some assoc Proto |
| $^{150}$Nd-Sm | *(p,t)* NP A195, 385 (1972)<br>*(t,p)* NP 86, 145(1966)<br>*(3He,n)* NP A269, 125 (1976) | The c |



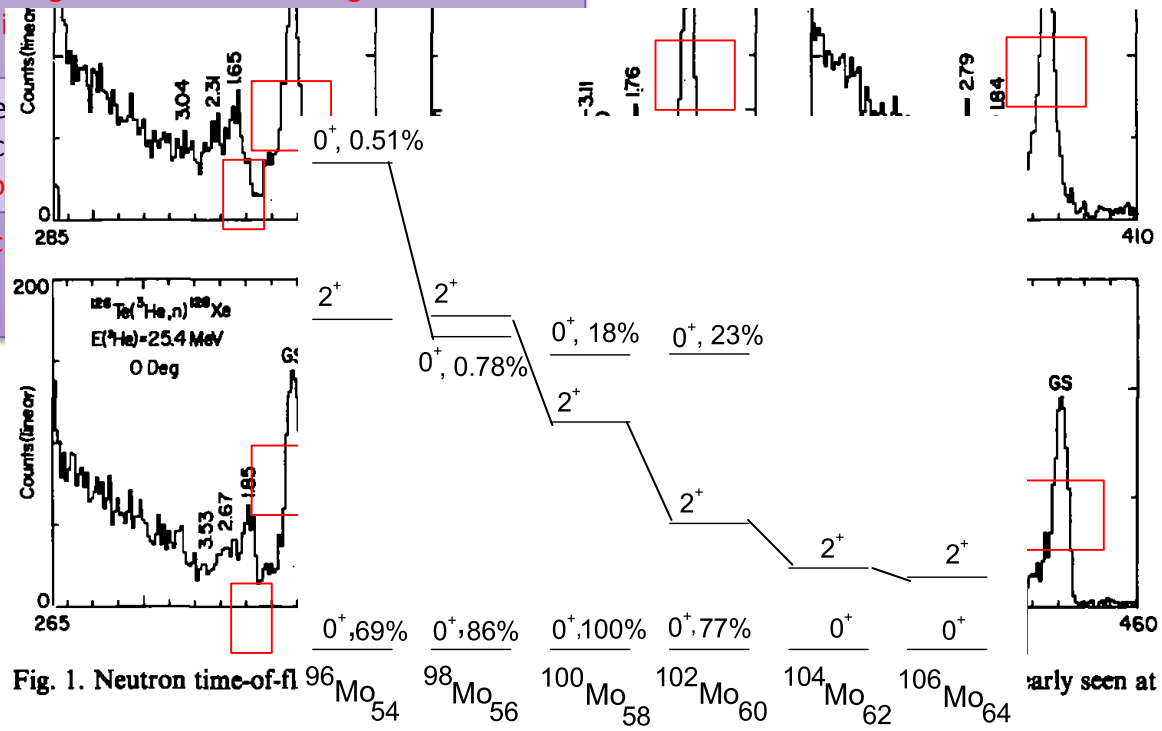




Fig. 1. Neutron time-of-fl ... early seen at

# Pairing and Double Beta Decay

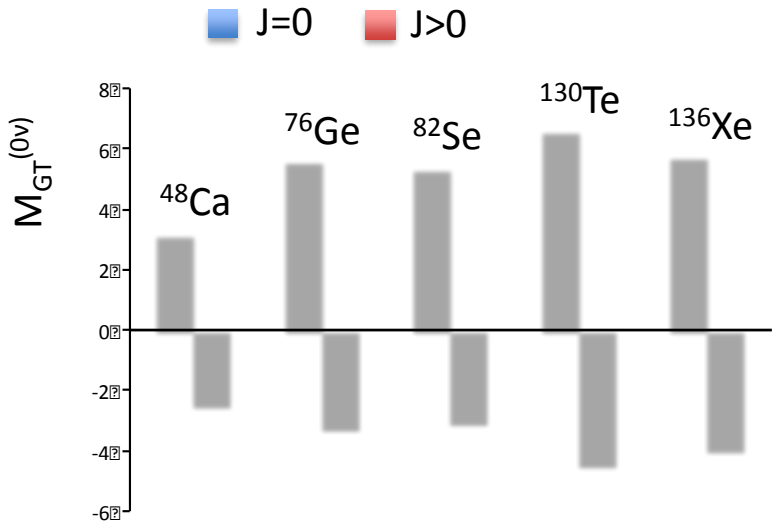
Many authors have noted the importance of contributions from J=0 nucleon pairs compared to J>0 in the matrix element of the 0ν mode.

Seems ubiquitous: appears in different theoretical treatments and across all decay candidates

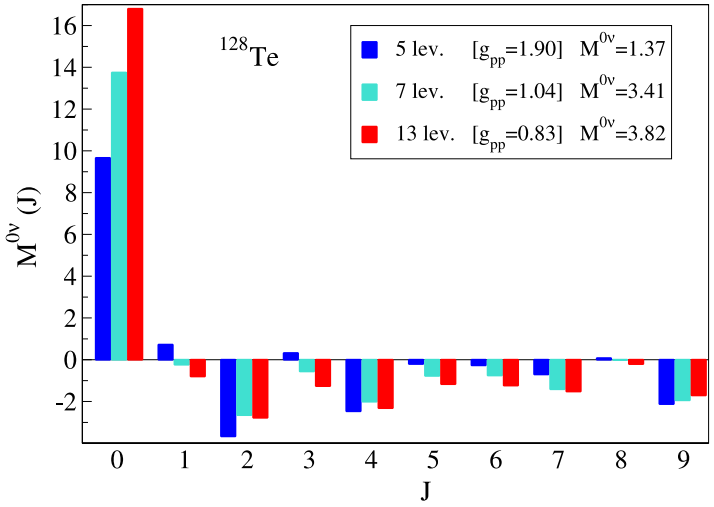

$$\hat{M}^{(0\nu)} = \sum_{J^\pi} \hat{P}^\dagger_{J_n^\pi} \hat{P}_{J_p^\pi}$$

A couple of recent examples:

Contributions to the GT matrix element with:




SM: *Caurier et al. PRL 100, 052503 (2008)*

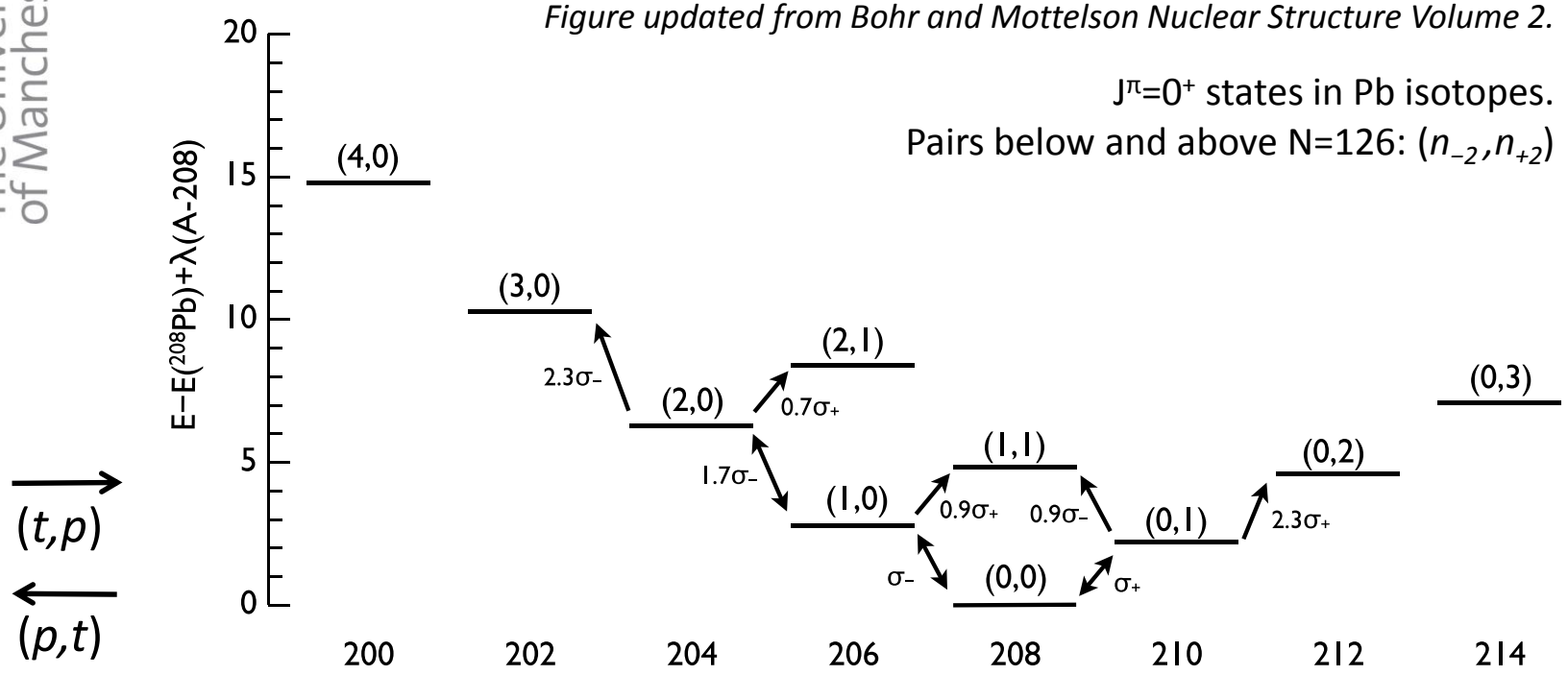



QRPA: *Escuderos et al. JPG 37, 125108 (2010)*

# Classical Example: N=126 Magic Gap

*Figure updated from Bohr and Mottelson Nuclear Structure Volume 2.*

$J^{\pi}=0^{+}$ states in Pb isotopes.
Pairs below and above N=126: $(n_{-2},n_{+2})$

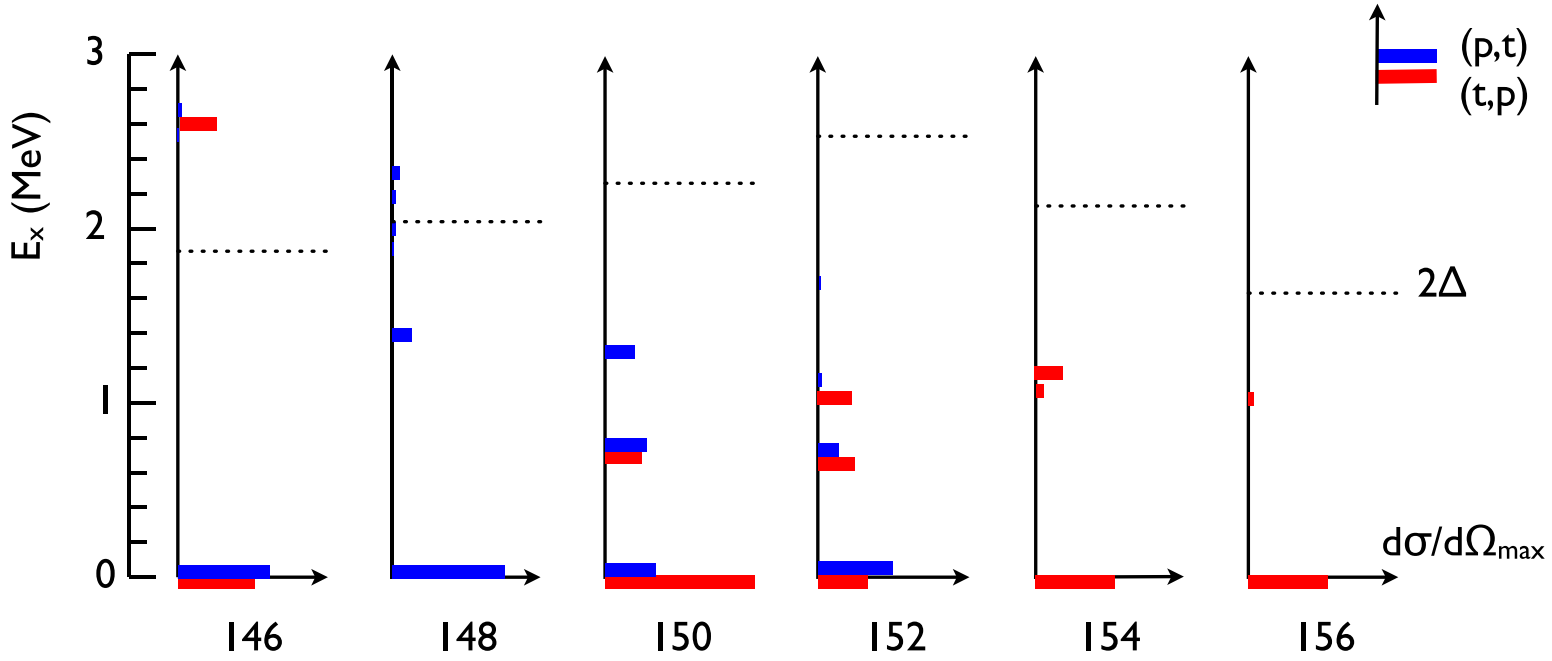


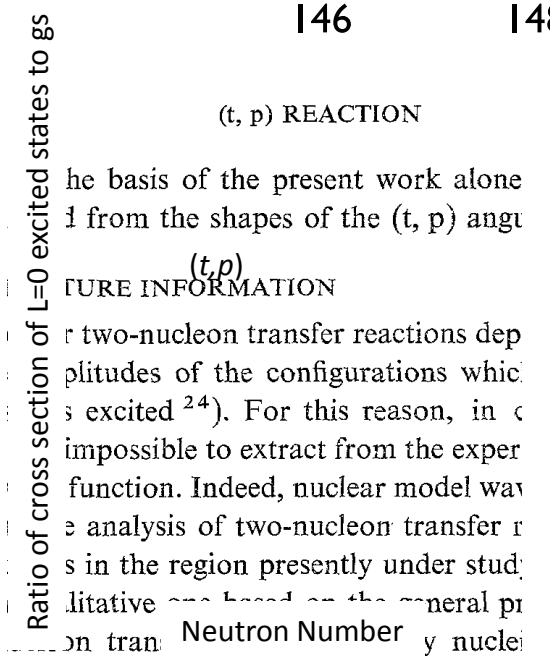

Large gap in neutron levels associated with N=126.
Pair addition and removal creates pairing vibrations relative to $^{208}$Pb "vacuum".
If pairs are identical and interactions between them can be neglected harmonic spectrum results:

$$E = \hbar\omega_{-2}n_{-2} + \hbar\omega_{2}n_{2}$$

Subshell gaps in spherical and Nilsson schemes also give rise to pairing vibrations; the gaps break the BCS symmetry.

# $^{150}$Nd-Sm:




(t, p) REACTION

he basis of the present work alone
l from the shapes of the (t, p) angu

(t,p)

TURE INFORMATION

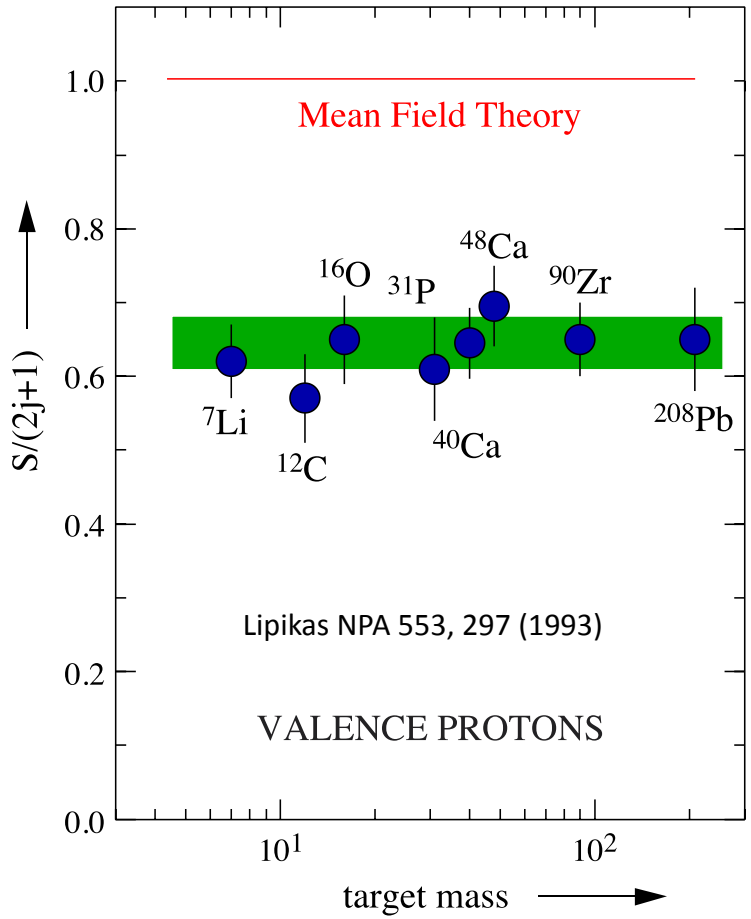
r two-nucleon transfer reactions dep
plitudes of the configurations whic
s excited [24]). For this reason, in
impossible to extract from the exper
function. Indeed, nuclear model wa
e analysis of two-nucleon transfer r
s in the region presently under stud
litative one based on the general pr
on tran y nuclei

- N≈90 Sm nuclei are the classical example of shape transition effects in pair transfer.
- Nd nuclei show globally similar effects in (*p,t*) and (*t,p*), although differs in the detail of the excited states.
- $^{148,150}$Nd($^{3}$He,*n*)$^{150,152}$Sm does not populate excited $0^{+}$ states.

*Sm Bjerregaard NP 86, 145 (1966), Debenham NP A195, 385 (1972.)*
*Nd Chapman NP A186, 603 (1972).*
*Nd protons Alford NP A321, 45 (1979).*

# Short Aside: Quenching of Cross Sections

Confirm and extends an effect identified in (e,e'p) studies at NIKEF in 1990's…




…now clear that it is independent of whether nucleon added or removed, type of nucleon transferred, nuclear mass, reaction type or angular momentum transfer…

…appears to be a uniform property in large part thought to be due to the effects of short-range correlations (SRC).

In what follows, *choose* to normalise the observed population of the valence orbitals to *(2j+1)* using the individually deduced quenching factors.

- Somewhat conventional; since uniform property should not matter.
- In some ways, makes them *relative* numbers.
- Internal normalisation adopted has some more global consistency.

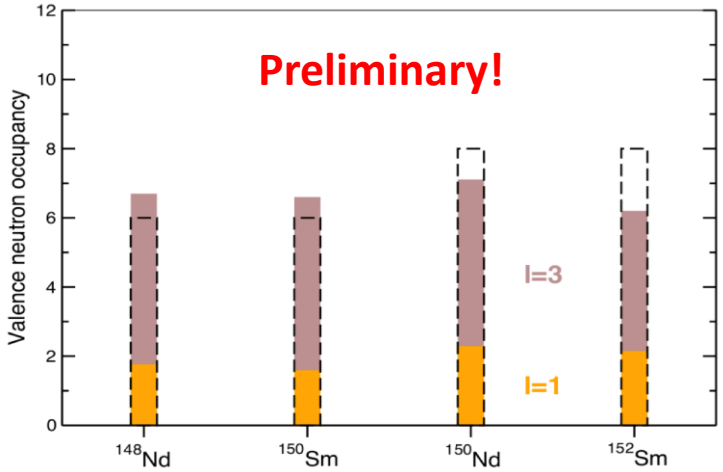
# Valence Nucleon Occupancies Relevant to 150Nd

Neutron adding and removal reactions on $^{148,150}$Nd and $^{150,152,154}$Sm targets.

Parent-product pair of putative 0νββ spans a well-known shape change between N=88 and 90.

Data taken on some neutron-transfer reactions (under analysis) and more experiments planned.

Early results suggest sum rules obeyed at least to 15%, but some puzzles to solve yet...




Facilities used (or will be used) at:

*What are the relative importance of different changes in orbital during the process?*

*($g_{9/2} \Rightarrow g_{9/2}$ versus $g_{9/2} \Rightarrow p_{3/2}$; or $0f_{5/2} \Rightarrow 0f_{5/2}$ versus $0f_{5/2} \Rightarrow 1p_{3/2}$).*

*How much do BCS pair-correlations matter?*

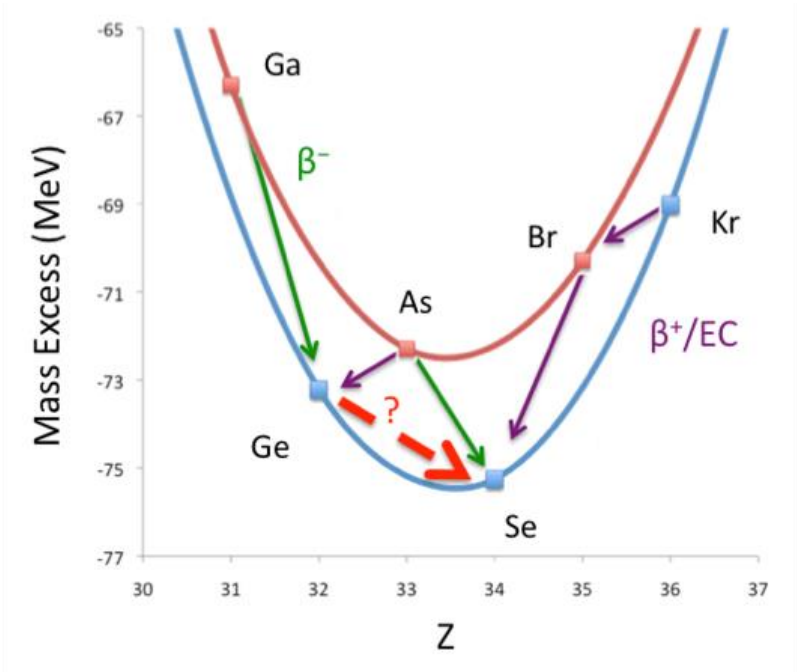
*How good is closure? (Only easily tested in QRPA approaches…how good are they?)*

*Renormalisation of operators is an unresolved issue: to quench or not quench $g_A$?*

# What is double beta decay?

Two neutrons, bound in the ground state of an even-even nucleus, transform into two bound protons, typically in the ground state of a final nucleus.

Observation of rare decays is only possibly when other radioactive processes don't occur…
a situation that sometimes arises naturally, usually thanks to pairing.




Accompanied with:

- Two electrons and two neutrinos (2νββ), observed in 10 species since 1987.
- Two electrons only (0νββ), for which a convincing observation remains to be made.

*[ Sometimes to excited bound states; sometimes protons to neutrons with two positrons; positron and an electron capture; perhaps resonant double electron capture. ]*

# Limits on 0νββ

| Isotope | Half Life (years) | Effective neutrino mass (eV) | |
|---|---|---|---|
| $^{76}$Ge | >2.1x10$^{25}$ | <0.25-0.62 | GERDA-1 |
| $^{100}$Mo | >1.1x10$^{24}$ | <0.34-0.87 | NEMO-3 |
| $^{130}$Te | >2.8x10$^{24}$ | <0.31-0.76 | CUORICINO |
| $^{136}$Xe | >1.9x10$^{25}$ | <0.14-0.34 | KamLAND-Zen |
| | >1.1x10$^{25}$ | <0.69-1.63* | EXO |

Taken from Barabash arXiv:1403.2870 (2014);
updated with EXO Nature 510 (2014) 229

# Measurements of 2νββ

| Isotope | Half life (year) |
|---|---|
| $^{48}Ca$ | $4.4(6)\times10^{19}$ |
| $^{76}Ge$ | $1.6(1)\times10^{21}$ |
| $^{82}Se$ | $0.92(7)\times10^{20}$ |
| $^{96}Zr$ | $2.3(3)\times10^{19}$ |
| $^{100}Mo$ | $7.1(4)\times10^{18}$ |
| $^{100}Mo$ ($0^+$) | $6.2(6)\times10^{20}$ |

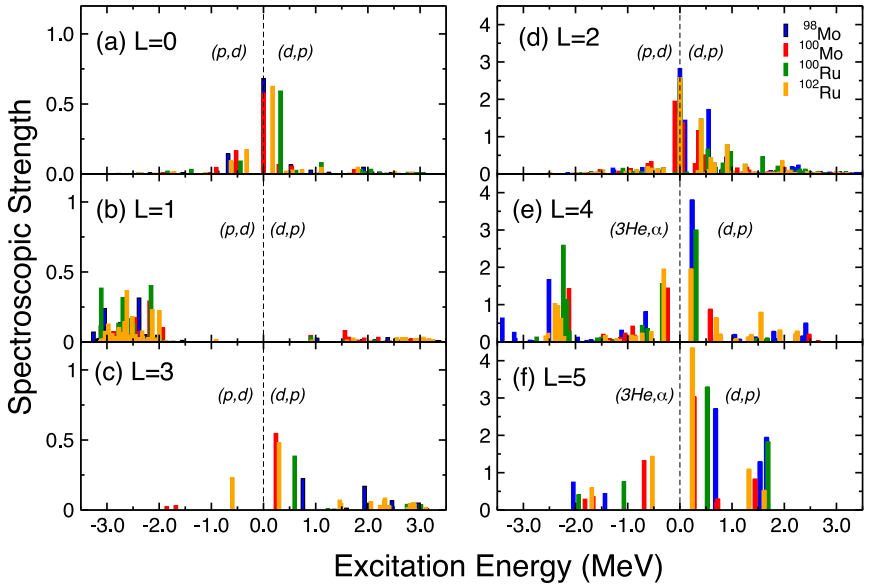
| Isotope | Half life (year) |
|---|---|
| $^{116}Cd$ | $2.85(15)\times10^{19}$ |
| $^{128}Te$ | $2.0(3)\times10^{24}$ |
| $^{130}Te$ | $6.9(13)\times10^{20}$ |
| $^{136}Xe$ | $2.2(1)\times10^{21}$ |
| $^{150}Nd$ | $8.2(9)\times10^{18}$ |
| $^{150}Nd$ ($0^+$) | $1.3(3)\times10^{20}$ |
| $^{238}U$ | $2.0(6)\times10^{21}$ |

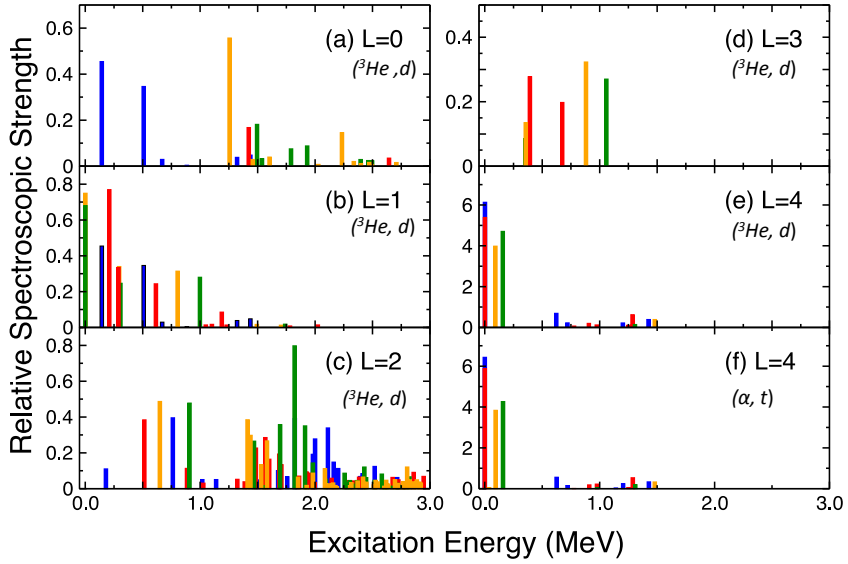
# Sources of Uncertainty

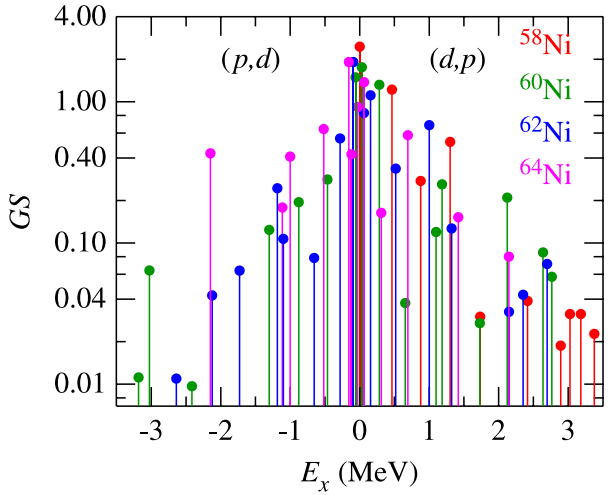
- *Cross sections:* constant aperture settings and use of Rutherford normalisation removes a lot of **systematic issues**, ones that remain are at the level of around 5%. **Statistics** are usually a few percent for most states.
- *Reaction modelling*: **DWBA parameter choices** can lead to large absolute variations (10-20% for sensible choices) but less in relative numbers, usually better than 5%. Other reaction models (ADWA,…) similar effects; DWBA is the most convenient and most applicable.
- Remaining uncertainties are difficult: (i) *unobserved strength* and (ii) *non-direct contributions*.

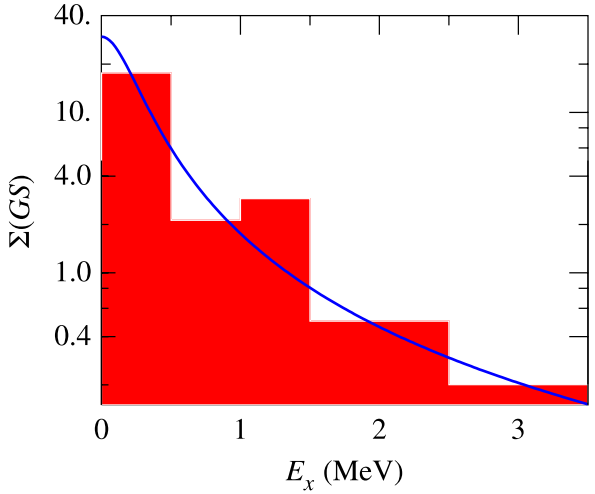
RMS deviation of summed occupancy compared to number of valence nucleons is generally of order 0.1 nucleon…estimates of (i) and (ii), as best you can, perhaps suggest errors are at the level of a few 0.1 nucleons.

# Missing strength?



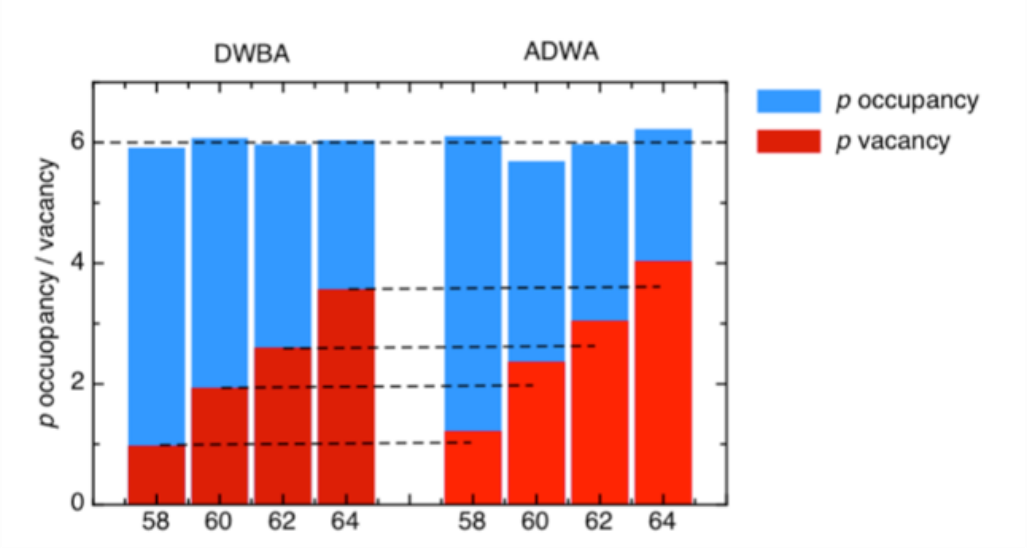

Missing < 2.8 % strength

# Non-direct processes?

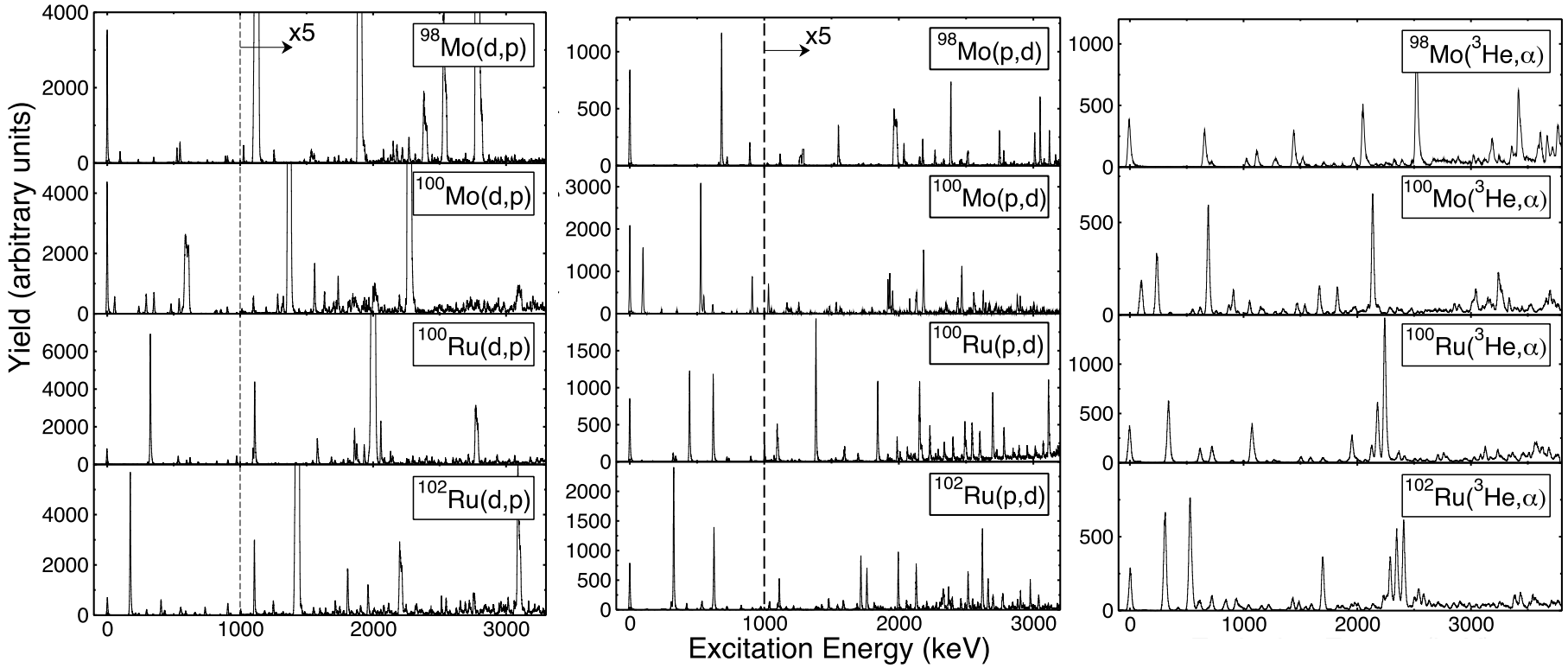
Non-direct processes likely to be more important for states populated weakly in the reaction. Is summing in many weak states introducing significant non-direct contributions?

For A=100 systems, difference between occupancies deduced from all states compared to occupancies deduced from states with SF > 1% is at most around 0.1 nucleons.

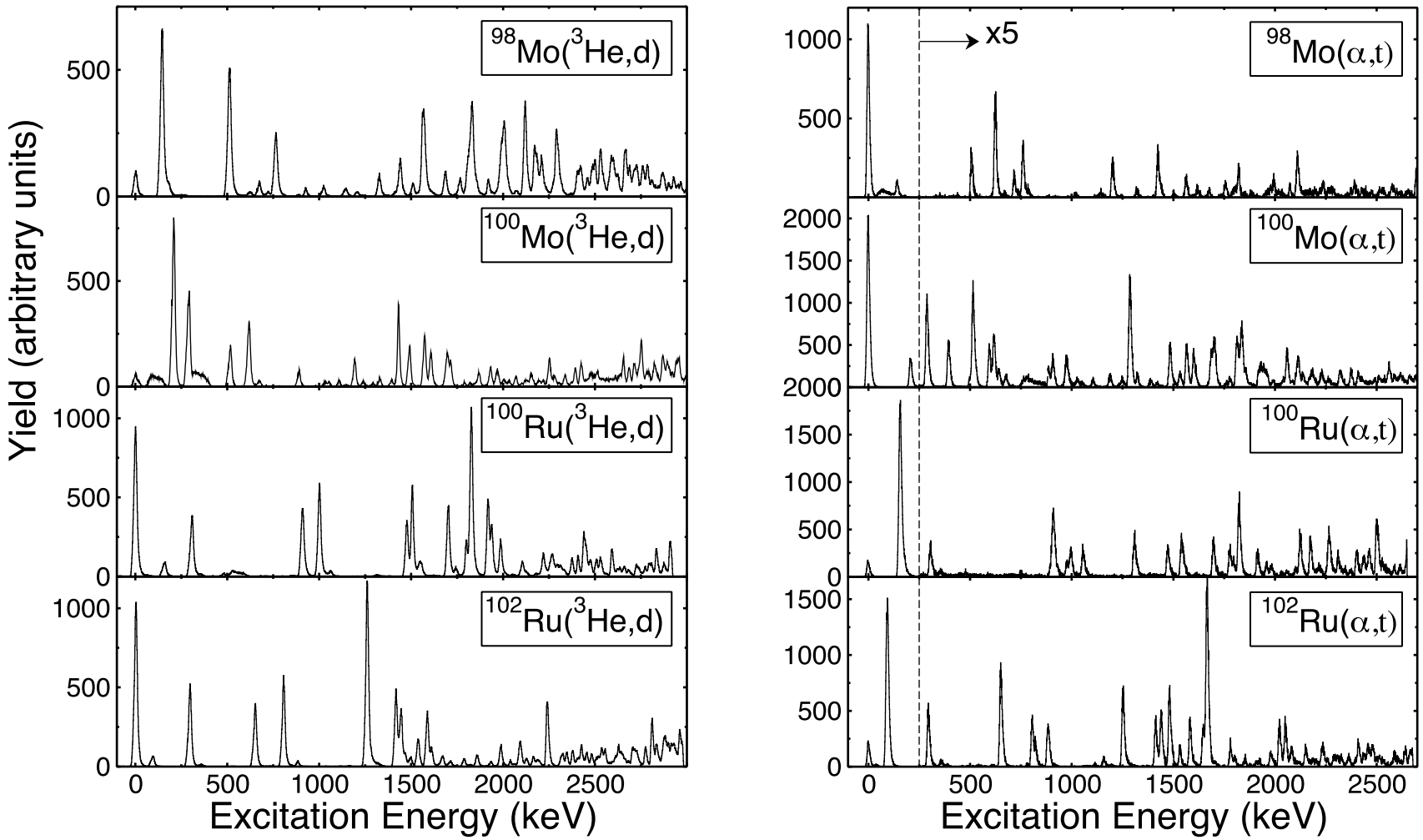
# ADWA


DWBA
ADWA
p occupancy
p vacancy
p occuopancy / vacancy
0
2
4
6
58
60
62
64
58
60
62
64

# A=100 Neutron Transfer Spectra

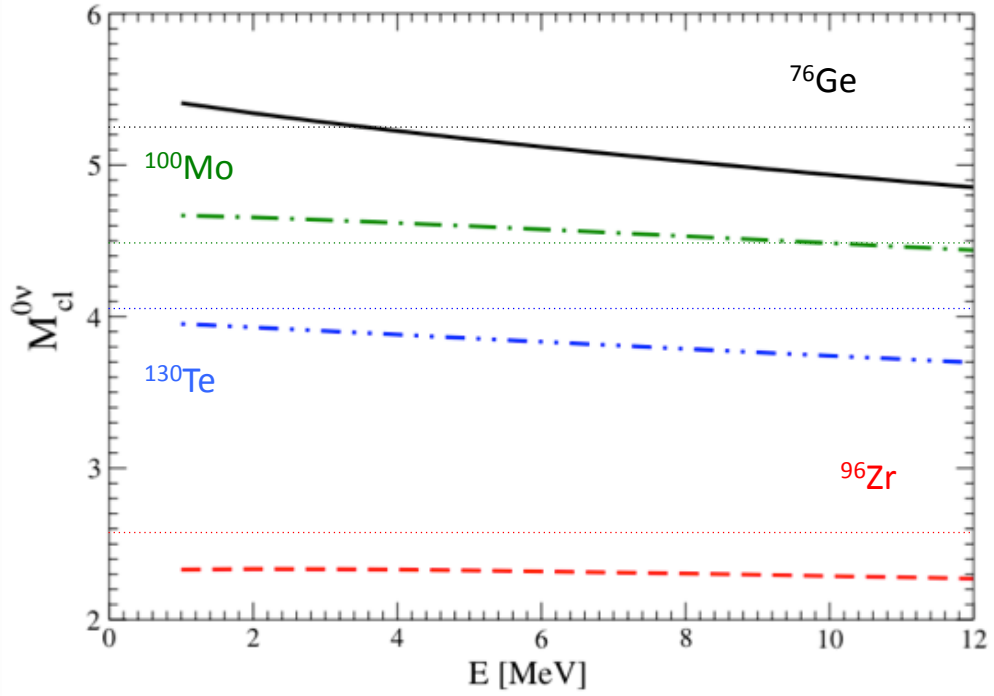


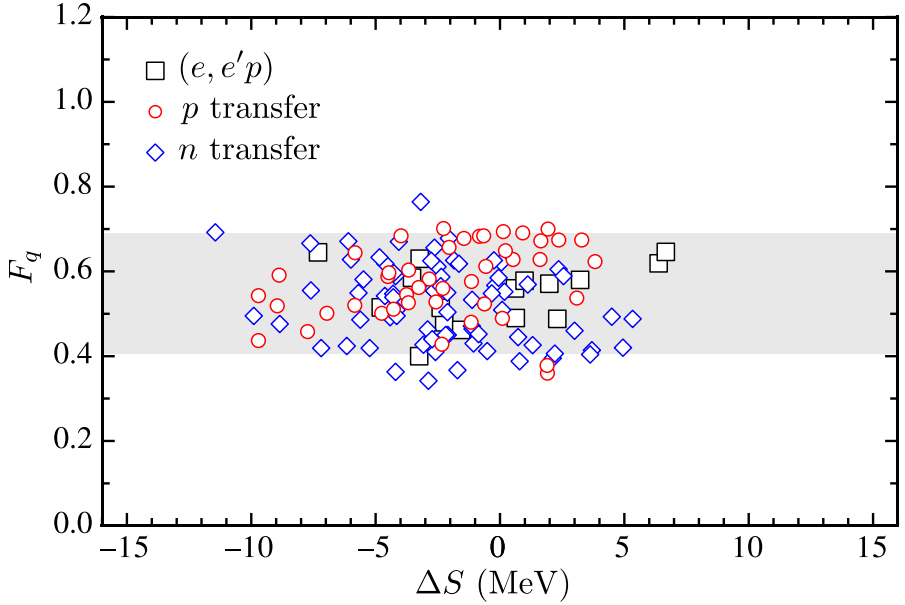
# A=100 Proton Transfer Spectra



# Tests of Closure

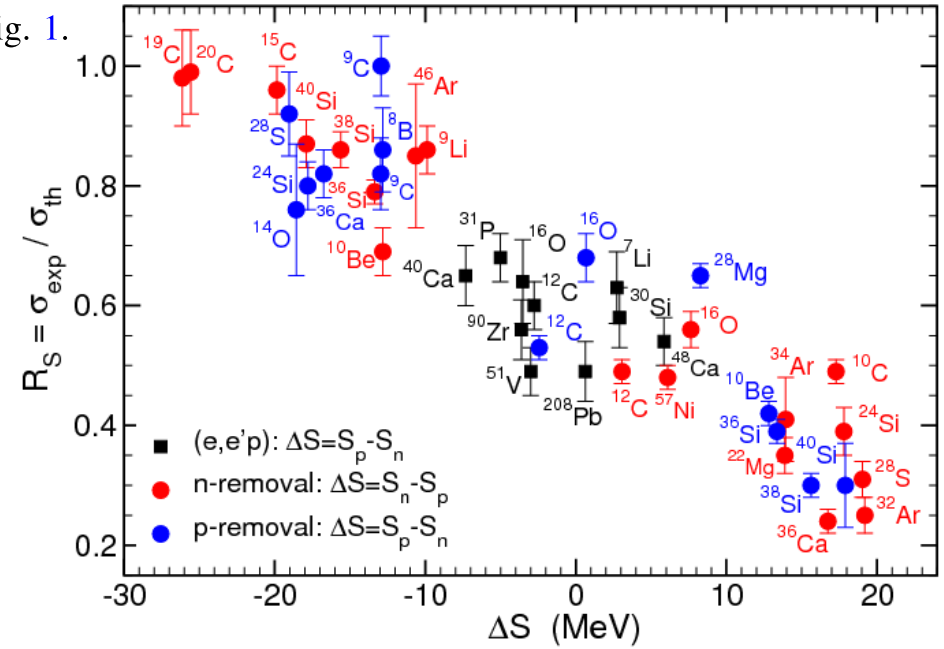
$M^{0\nu}$ calculated using closure within QRPA as a function of the assumed average excitation energy, compared to values without the approximation



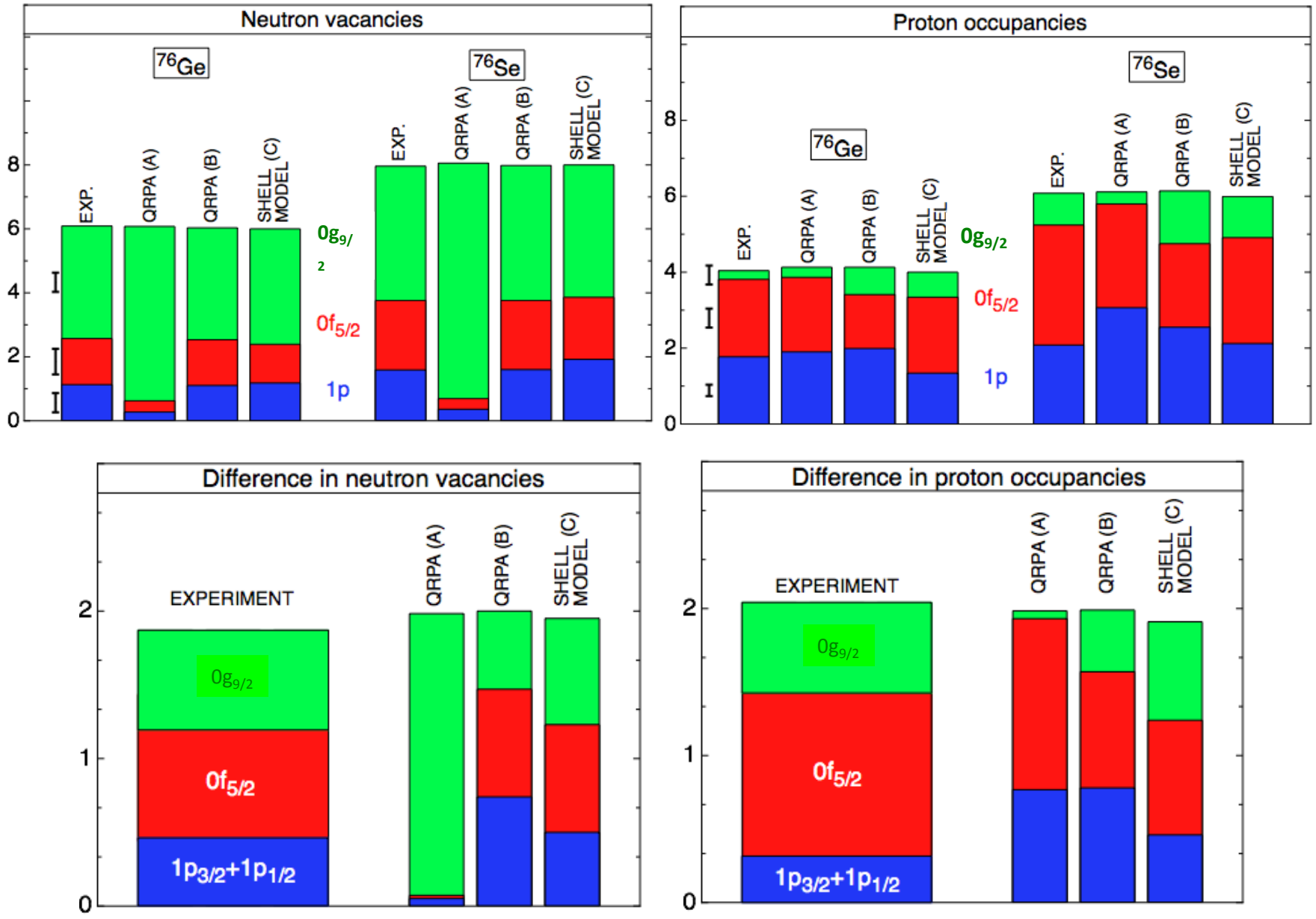



Quenching as a function of binding

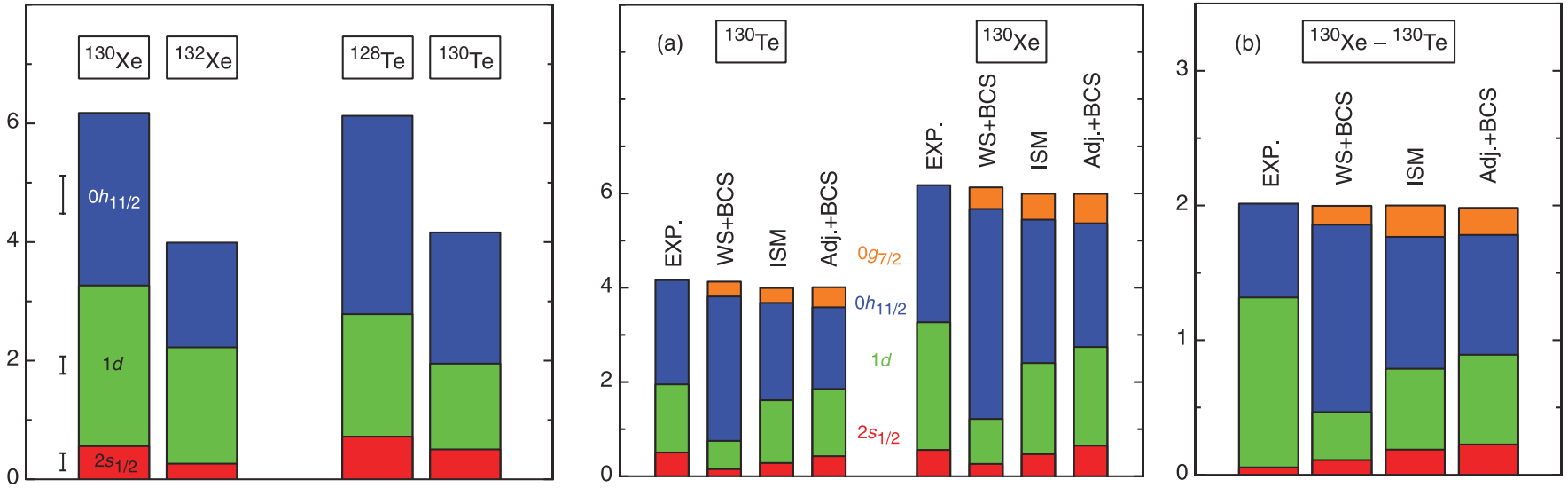
FIG. 3 (color online). $F_q$ versus $\Delta S$ for all of the data shown in Fig. 1. The grey band represents the $(e,e'p)$ data as in Fig. 1.



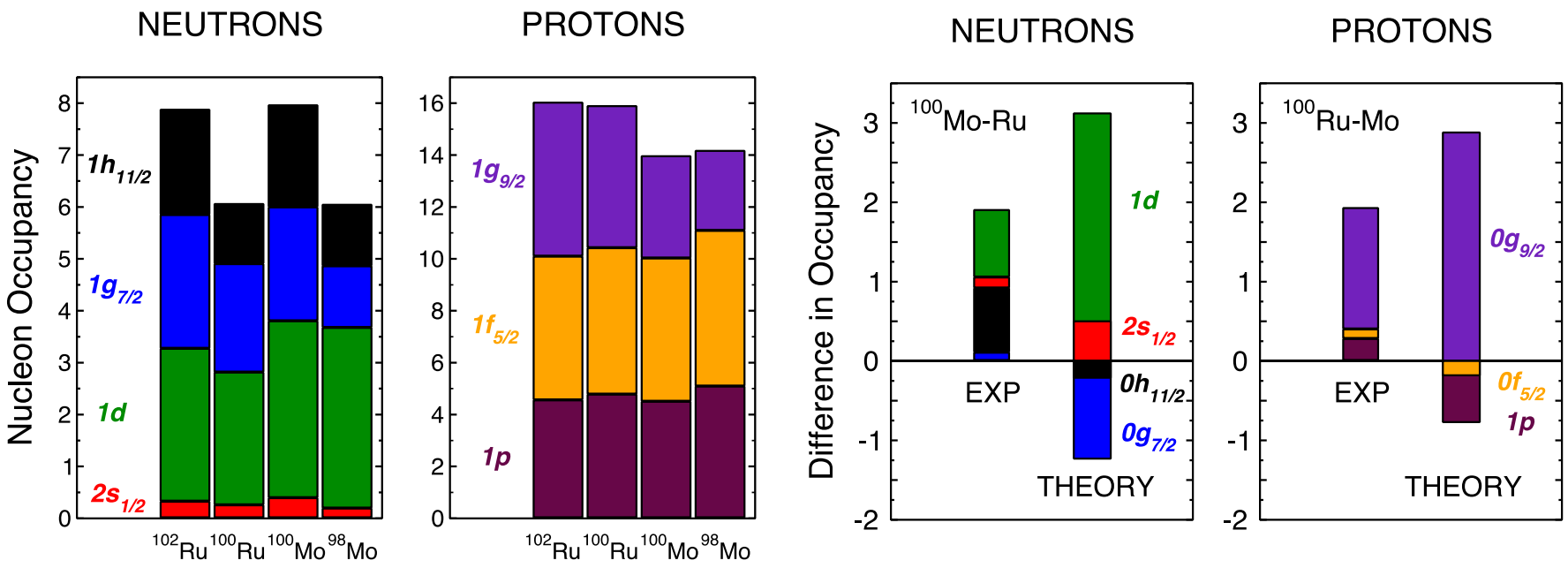
# Occupancies for $^{76}Ge$



# *Occupancies for $^{130}$Te*



# *Occupancies for $^{100}$Mo*



# Calculating Nuclear Matrix Elements

Need to know the wave functions of the nuclei involved.

For the foreseeable future, it will not be possible to calculate the properties of large nuclei using QCD!

Current "best" nuclear-structure calculations take empirical NN interactions, corrected for many-body forces, and solve the many-body Schrödinger equation directly using Monte-Carlo techniques, giving accurate wave functions as heavy as A≈12.

For candidate double beta decay nuclei, have to resort to something "messier":

<u>Mean-field theory</u>:
independently moving nucleons; anti-symmetrised determinant of spin-coupled independent particles

<u>With substantial correction</u>:
Add correlations between nucleons, using *QRPA* or *shell model*.
(Some insights from other approaches such as Interaction Boson Model…)

# Quasiparticle Random-Phase Approximation

(and very *many* variants).

PLUS! Treats many nucleons as "active" and allows them to move in a large single-particle space.

MINUS! Can only cope with specific correlations of simple type (actually best suited for describing collective excitations).

METHOD: small deviations from the unperturbed ground state by bosonizing Hamiltonian and transition operators (RPA). *Pairing-like correlations taken into account using BCS theory (QRPA).*

ISSUES: 2νββ decay rate is VERY sensitive to particle-particle coupling strength; small change results in large fluctuations and the RPA approx collapses. VERY many variants to try to get around this issue ('second' QRPA, RQRPA, BCS-RQRPA….), but none is clearly superior to the others.

ALL leave out (non-RPA!) correlations.

ALL have some truncation of model space so effective operators are needed.

# Shell Models.

PLUS! Allows arbitrary correlations between nucleons.

MINUS! Can only cope with a small number of "active" nucleons: severely truncated.

METHOD: Diagonalize empirically-deduced effective interactions within a restricted model space. Could get arbitrary accuracy, provided a large enough model space is used; current computer technology allows bases of millions of states, but this barely sufficient in heavy systems.

ISSUES: Nucleon-nucleon interaction in a restricted basis needs modification, and effective interactions are usually based on fitting to "simple" configurations in "simple" nuclei.

Any truncation of model space requires effective operators to account for effects of nucleons outside of the model space.

# Theoretical Prospects
# (as viewed by an experimentalist!)

Despite everything, most "good" calculations agree to within a factor of a few.

Attempts are made to judge uncertainty from the range of calculations!

Increases in computing power guarantee that shell-model calculations will get better and better; although issues with renormalization of effective operators for double beta decay have not yet been fully dealt with.

Can help by *measuring* related observables (in addition to 0νββ searches!):

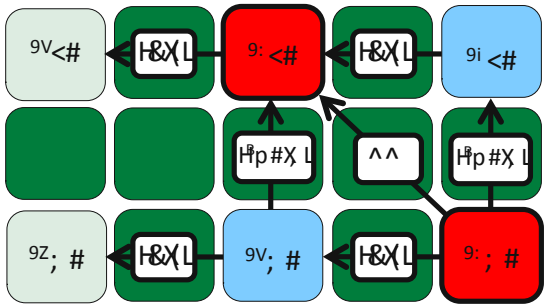
- The more observables a calculation can reproduce, in general, the more trust you can put in it.
- If there are free parameters, other observables might be used to fix them.
- If renormalization is needed, other observables might be used to do it.

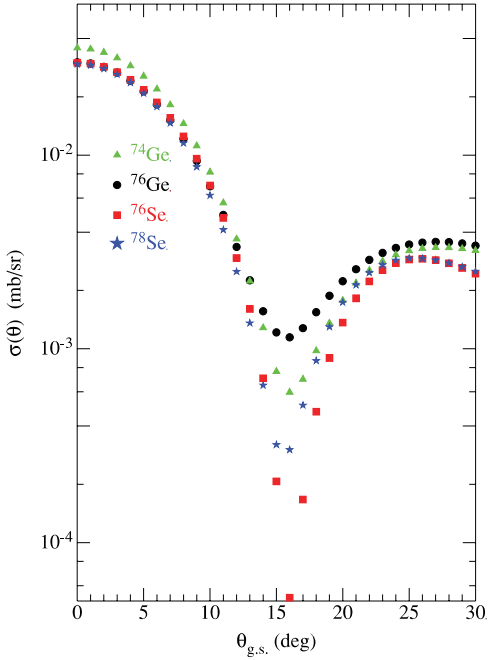
And need to build a deeper intuition about what is important…

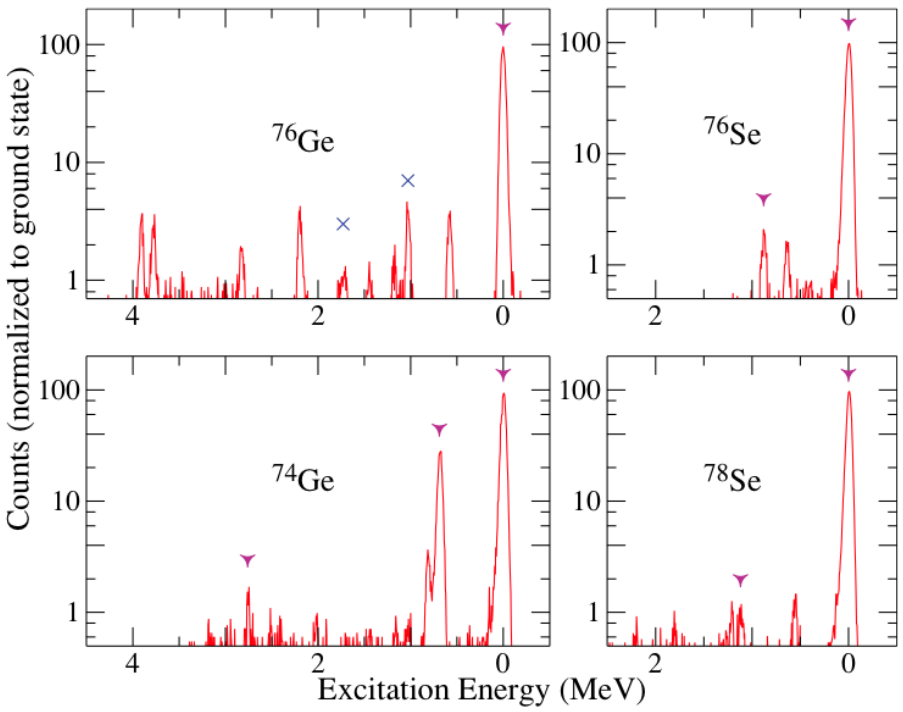
……"how much related?"

# Neutron Pair Transfer: $^{76}$Ge/$^{76}$Se

Reactions involving transfer of a pair of nucleons between simple $0^+$ BCS wave functions are enhanced.





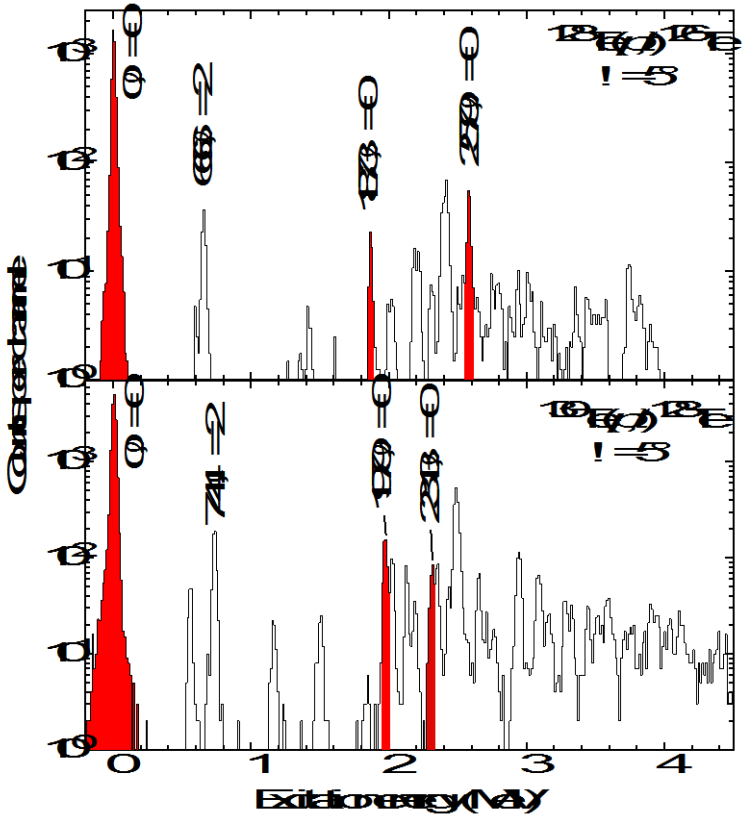




For $^{76}$Ge and $^{76}$Se, (p,t) strength is predominately connects the $0^+$ ground states, indicating they are simple BCS paired states. They have quantitatively similar pair correlations, significantly simplifying calculations.
In other Ge nuclei, things are not so fortuitously simple.

($^3$He,n) reactions to pursue *proton*-pairing properties is technically much more demanding; Notre Dame Fall 2011.

# Pair Transfer: $^{130}$Te/$^{130}$Xe

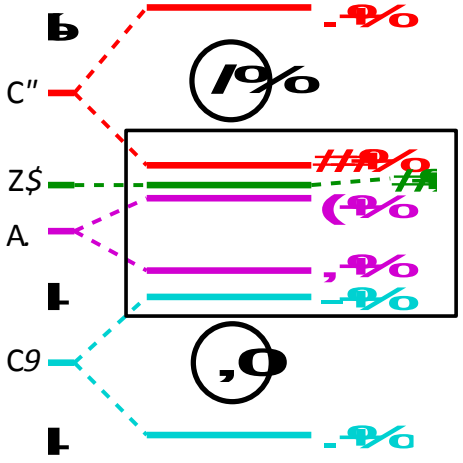
*(p,t): T. Bloxham et al. PRC 82, 027308 (2010)*
*($^{3}$He,n): W.P. Alford et al. NPA 323,339 (1979)*



| Reaction | E (MeV) | σ (mb/sr) | Ratio[a] | Normalized strength[b] |
|---|---|---|---|---|
| $^{128}$Te(p,t) | 0 | 4.21 | 90 | 1.21 |
| | 1.873 | 0.06 | 20 | 0.02 |
| | 2.579 | 0.15 | 21 | 0.04 |
| $^{130}$Te(p,t) | 0 | 3.49 | 89 | 1.00 |
| | 1.979 | 0.05 | 50 | 0.01 |
| | 2.313(4)[c] | 0.05 | >20 | 0.01 |
| $^{128}$Te($^{3}$He,n) | 0 | 0.24 | – | 0.96 |
| | 2.13 | 0.095 | – | 0.32 |
| $^{130}$Te($^{3}$He,n) | 0 | 0.26 | – | 1.00 |
| | 1.85 | 0.098 | – | 0.34 |
| | 2.49 | 0.062 | – | 0.21 |

*Neutron-pairing properties:* simple as with Ge/Se.

*Proton-pairing properties:* existing data is inconsistent with QRPA assumptions.
Classic *pairing-vibrational state* with splitting BCS strength, probably due to Z=64 subshell gap.



Single-proton levels.

# Double beta decay with neutrinos, 2νββ

Rate of the process:

$$\left[T_{1/2}^{2\nu}\right]^{-1} = G^{2\nu} \quad |M^{2\nu}|^2$$

Phase space factors ~$Q^{11}$, weak coupling constants, Coulomb effects

Nuclear matrix element:

$$M^{2\nu} = M_{GT}^{2\nu} + \left(\frac{g_V}{g_A}\right)^2 M_F^{2\nu}$$

Fermi part:

$$M_F^{2\nu} = \sum_c \frac{\langle f|\tau|c\rangle\langle c|\tau|i\rangle}{(2E_c - E_i - E_f)/2}$$

GT part:

$$M_{GT}^{2\nu} = \sum_c \frac{\langle f|\sigma\tau|c\rangle\langle c|\sigma\tau|i\rangle}{(2E_c - E_i - E_f)/2}$$

Sum over states in the intermediate nucleus: e.g. $^{76}$As for $^{76}$Ge decay to $^{76}$Se

Amplitude for $\beta^-$ decay or (p,n) reaction

Amplitude for $\beta^+$ decay or (n,p) reaction

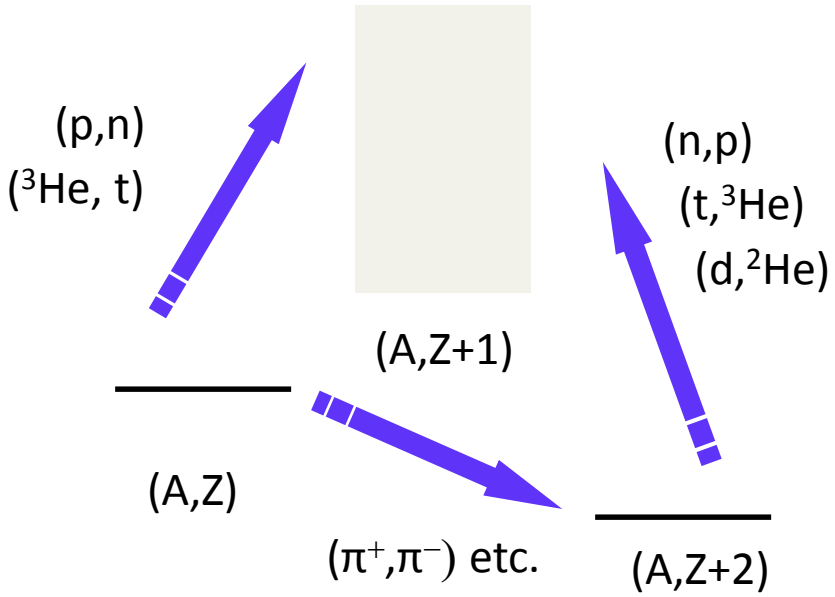
# GT Distributions and Charge exchange reactions

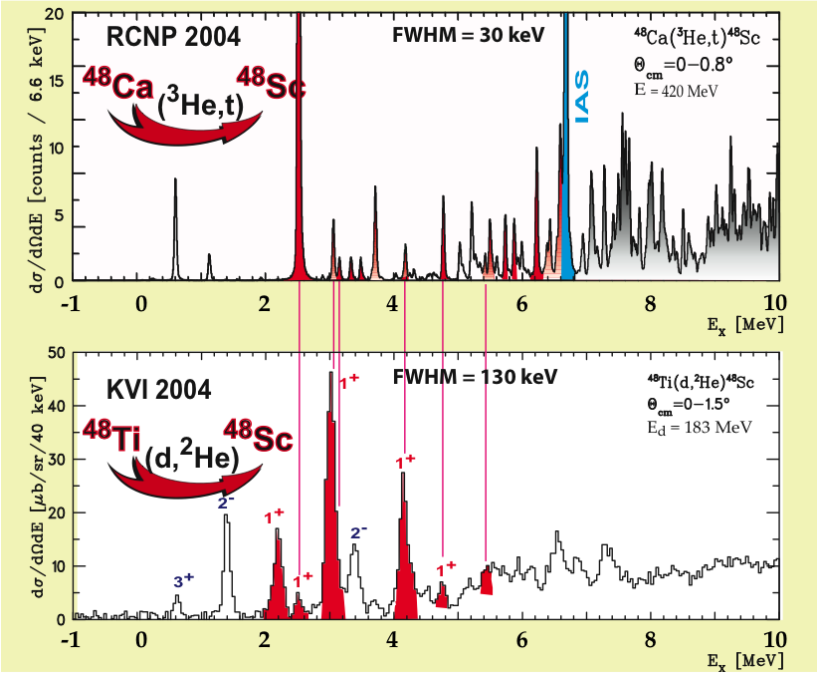
Forward-angle, single-charge exchange reactions (L=0) are sensitive to GT distributions populating $1^+$ states in intermediate system.

Very useful for checking calculations of 2νββ

For L≠0 transitions important for 0νββ are complex in hadronic reactions; no simple relationship exists to convert cross sections into the relevant strength distributions. Also relative phases of amplitudes not directly measurable!

Double charge exchange, e.g. ($\pi^+$,$\pi^-$), ($^{11}$Be,$^{11}$Li), convert two neutrons into protons but different operators involved AND complex reaction mechanisms.



# Neutrinoless double beta decay, 0νββ

Mediation by a virtual neutrino gives different features:

$$q \sim \hbar/r_{nn} \sim 50 - 100\mathrm{MeV/c}$$

A: Energy of intermediate excited states can be large up to several tens of MeV *(compare with few MeV for 2νββ).*

B: Angular momentum transfer is also large, up to 7-8 *ħ* (compare with *1ħ for 2νββ).*

$$M^{0\nu} \approx \langle f| \sum_{lk} H(r_{lk}, E)\tau_l\tau_k \left(\sigma_l\sigma_k - \left(\frac{g_V}{g_A}\right)^2\right) |i\rangle$$

Both F and GT transitions.

"Neutrino potential": depends on position of nucleons and (weakly) on the energy of intermediate state, due to A can replace by average (CLOSURE).

When expanding H into multipoles expect contributions up to 7-8.