Single Particle Levels and ββ decay matrix elements in the Interacting Boson Model

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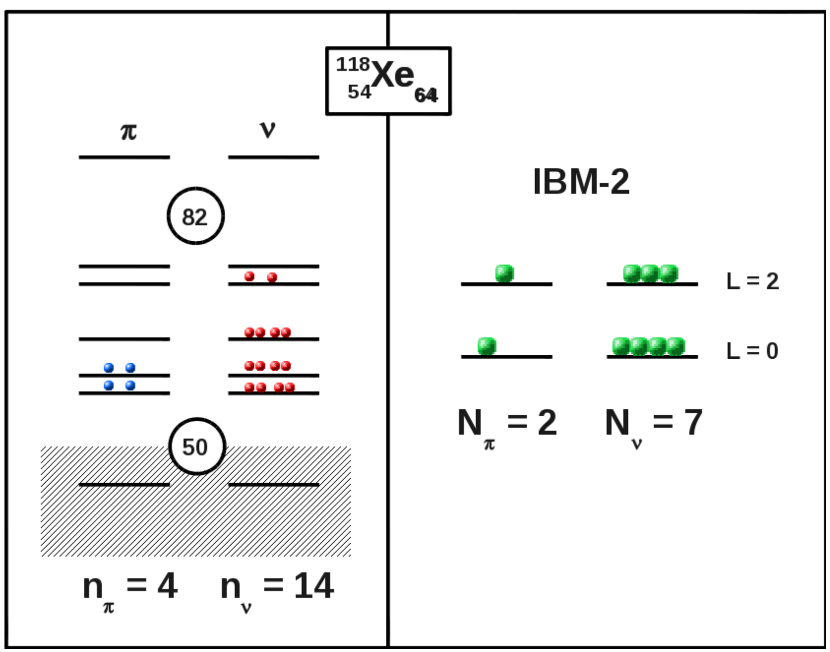
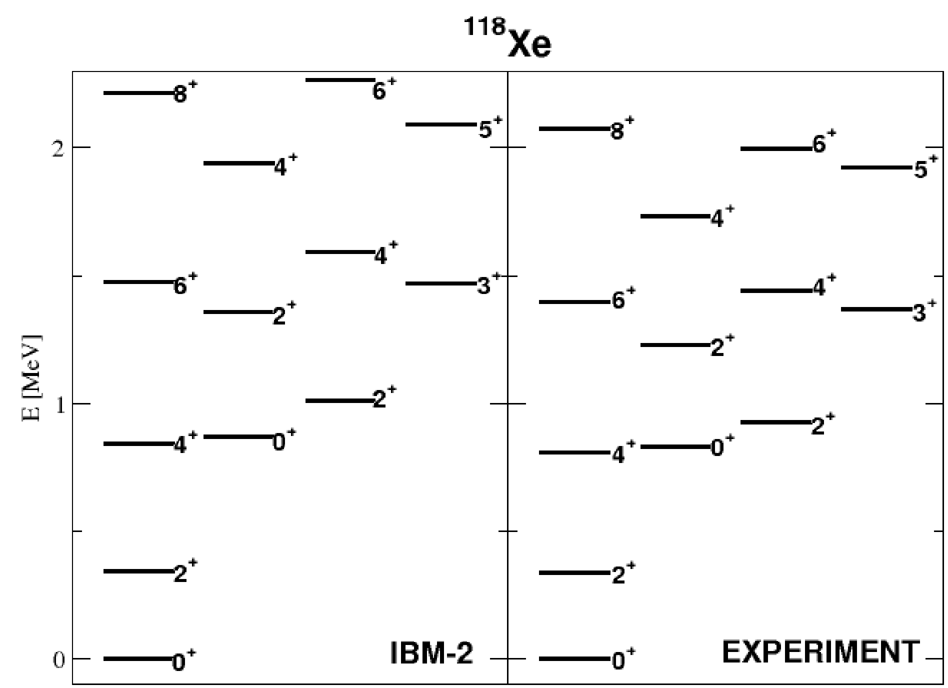
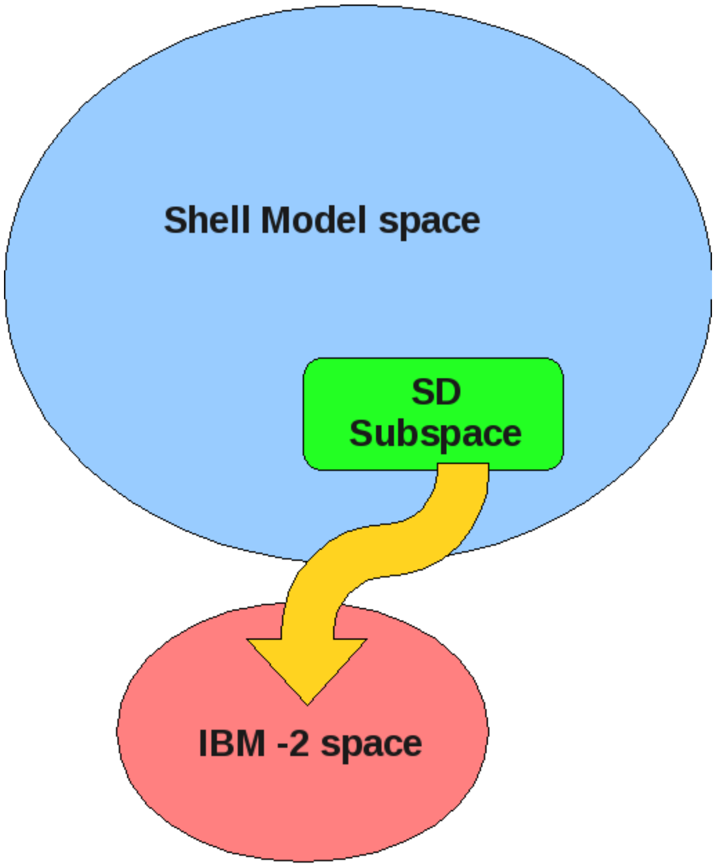
The Interacting Boson Model

$$\begin{aligned}
H_{IBM2} &= H_\pi + H_\nu + \kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)} + M_{\pi\nu}\left(\xi_1, \xi_2, \xi_3\right) \\
H_\rho &= \epsilon \hat{n}_{d_\rho} + \sum_{L=0,2,4} c_L^\rho \left(d_\rho^\dagger d_\rho^\dagger\right)^{(L)} \cdot \left(\tilde{d}_\rho \tilde{d}_\rho\right)^{(L)} \\
Q_\rho^{(2)} &= \left(s_\rho^\dagger \times \tilde{d}_\rho + d_\rho^\dagger \times \tilde{s}_\rho\right)^{(2)} + \chi_\rho \left(d_\rho^\dagger \times \tilde{d}_\rho\right)^{(2)}
\end{aligned}$$





IBM-2 and Shell Model



$$S_+ = \sum_j \alpha_j \frac{\sqrt{j+\frac{1}{2}}}{2} \left(C_j^\dagger C_j^\dagger\right)_0^{(0)}$$
$$D_+ = \sum_{j\leq j'} \frac{\beta_{jj'}}{\sqrt{1+\delta_{jj'}}} \left(C_j^\dagger C_{j'}^\dagger\right)_M^{(2)}$$

$$\begin{array}{rcl} |2N,0,0,0\rangle & \rightarrow & \left|s^N\right\rangle \\ |2N+1,1,j,m\rangle & \rightarrow & \left|s^N j\right\rangle \\ |2N+2,2,2,M\rangle & \rightarrow & \left|s^N d\right\rangle \\ & \ldots & \end{array}$$

$$O_{SM} \rightarrow o_{IBM} = \sum_k \gamma_k f_k\left[s,d,j\right]$$

$$\langle F\left|O_{SM}\right|I\rangle = \langle f\left|o_{IBM}\right|i\rangle = \left\langle f\left|\sum_k \gamma_k f_k\left[s,d,j\right]\right|i\right\rangle$$

Structure of the S and D pairs

- **BCS method + NOA approximation**

$$\frac{\alpha_j}{\sqrt{\Omega_e}} = \frac{v_j}{\sqrt{N_B}}, \quad \Omega_e = \sum_j \alpha_j^2 \left(j + \tfrac{1}{2}\right)$$

$$\beta_{jj'} = \frac{\langle j \| r^2 Y^{(2)} \| j' \rangle \left(u_j v_{j'} + v_j u_{j'}\right)}{K_\beta}, \quad \sum_{j \le j'} \beta_{jj'}^2 = 1$$

- **SDI method**

$$\left|0_1^+\right\rangle = \sum_j A_j \left|j^2; 0\right\rangle \to \alpha_j = \sqrt{\frac{\sum_j \left(j + \frac{1}{2}\right)}{j + \frac{1}{2}}} A_j$$

$$\left|2_1^+\right\rangle = \sum_{j \le j'} B_{jj'} \left|jj'; 2\right\rangle \to \beta_{jj'} = B_{jj'}$$

Occupancies in the IBM-2

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Occupation probabilities of single particle levels using the microscopic interacting boson model: Application to some nuclei of interest in neutrinoless double-β decay

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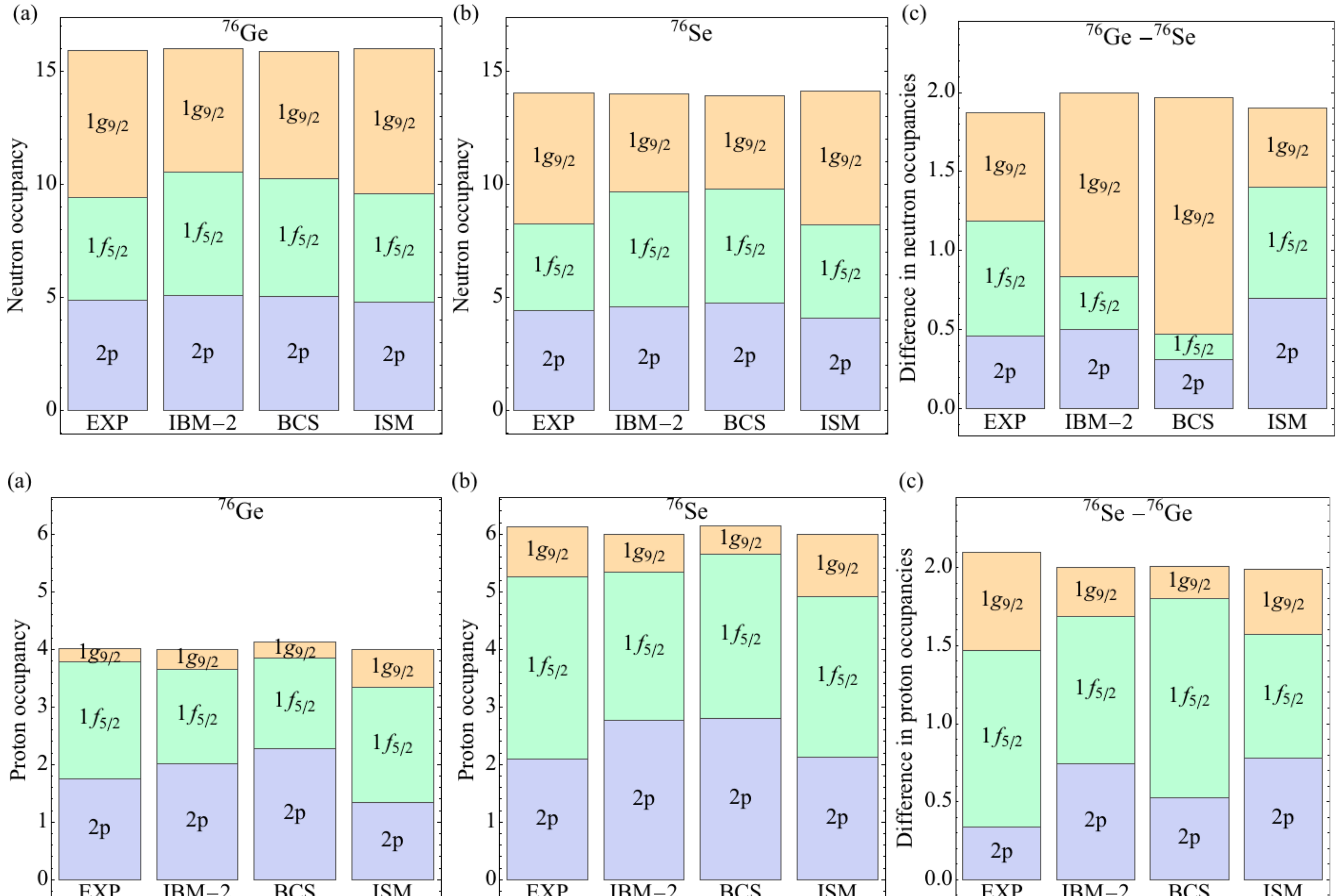
$$\hat{n}_j = C_j^\dagger \cdot \tilde{C}_j \mapsto \hat{n}_j^B = A_j s^\dagger \cdot \tilde{s} + B_j d^\dagger \cdot \tilde{d}$$

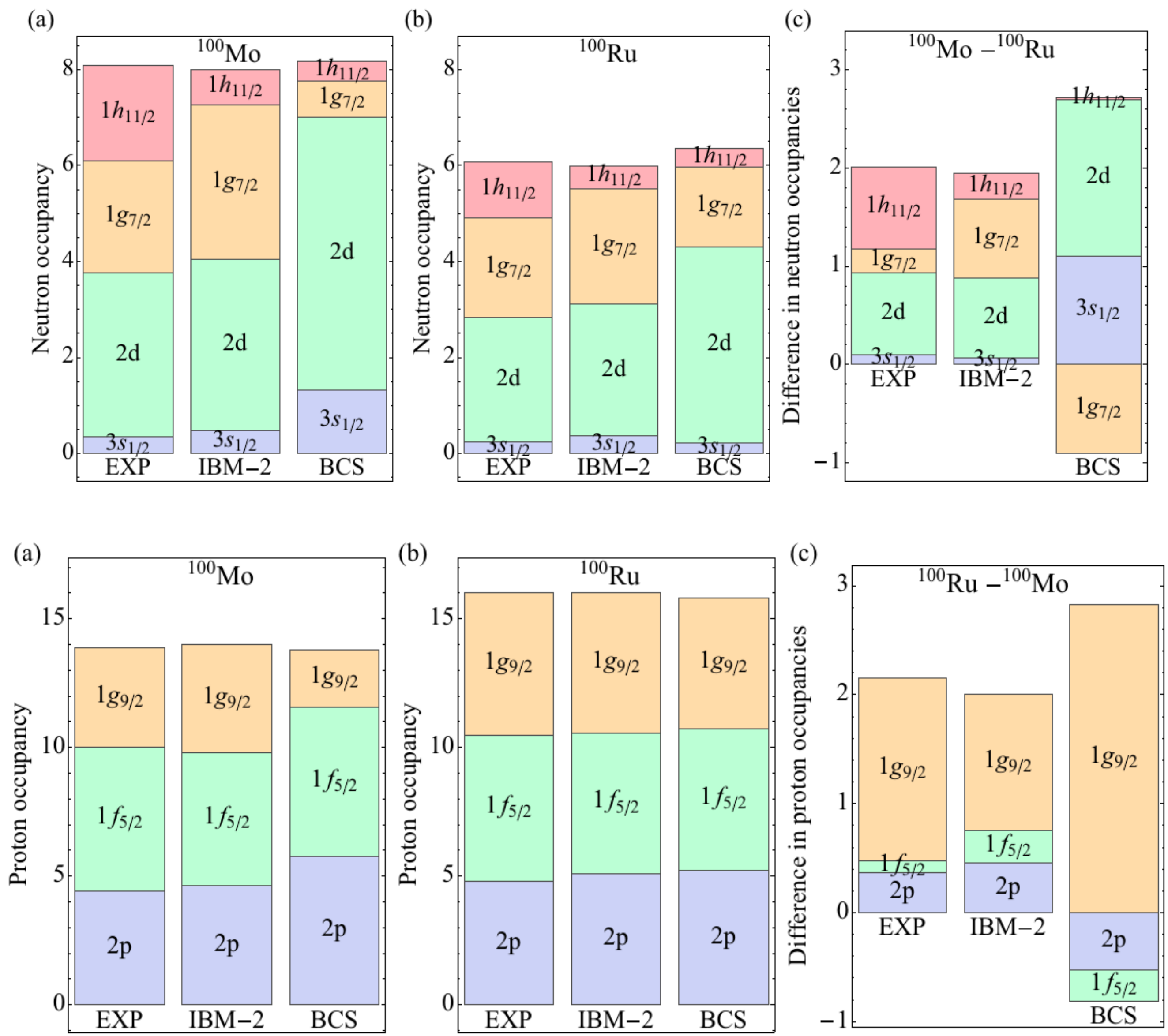
$$\begin{aligned} A_j &= \langle 2N,0,0 \, \| \hat{n}_j \| \, 2N,0,0 \rangle \\ B_j &= \frac{1}{\sqrt{5}} \langle 2N,2,2 \, \| \hat{n}_j \| \, 2N,2,2 \rangle \\ &- \left(1 - \frac{1}{N}\right) \langle 2N,0,0 \, \| \hat{n}_j \| \, 2N,0,0 \rangle \end{aligned}$$

Updated Single Particle Energies

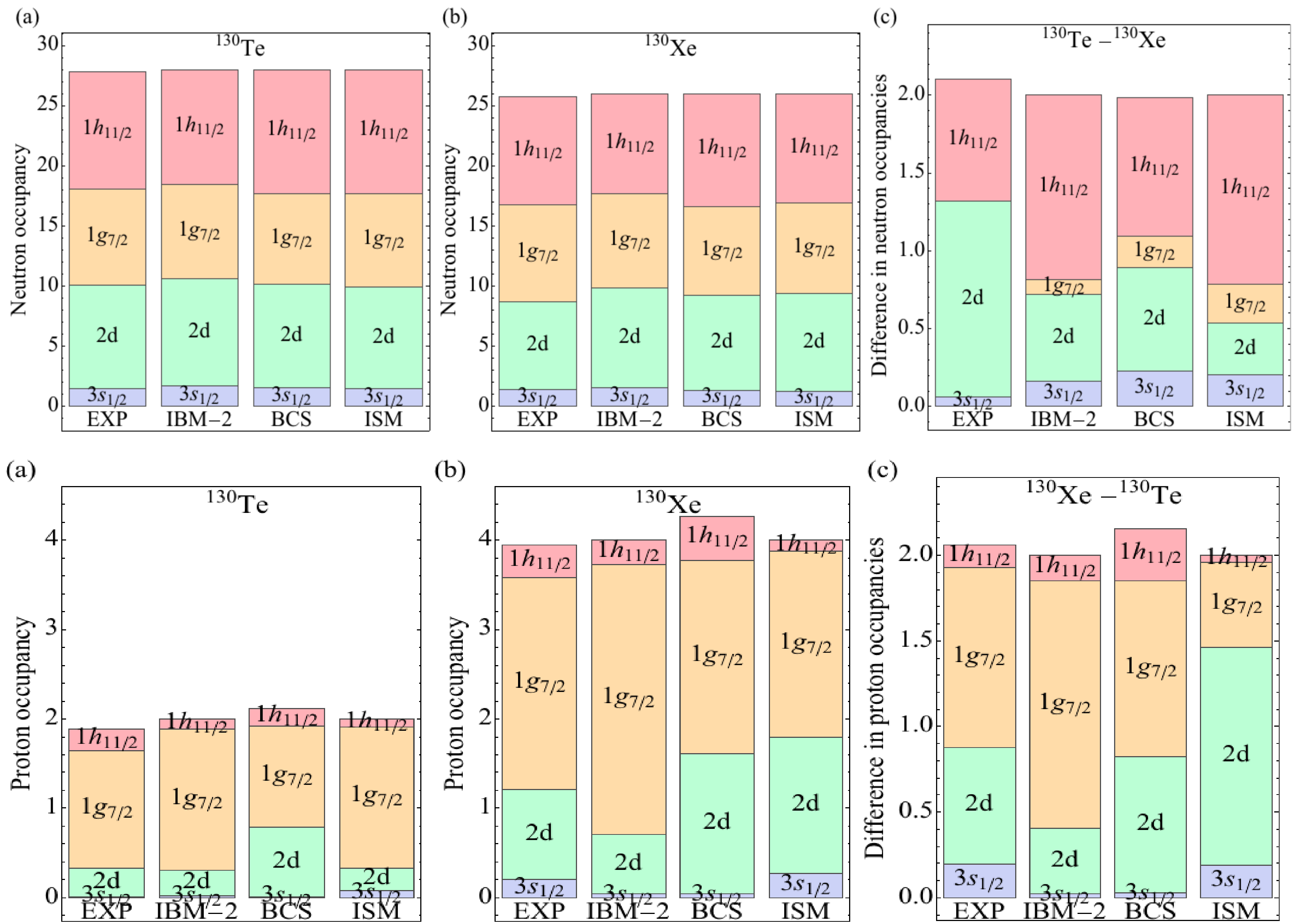
Occupancies for A = 76



Occupancies for A = 100



Occupancies for A = 130



Double beta decay in the IBM-2

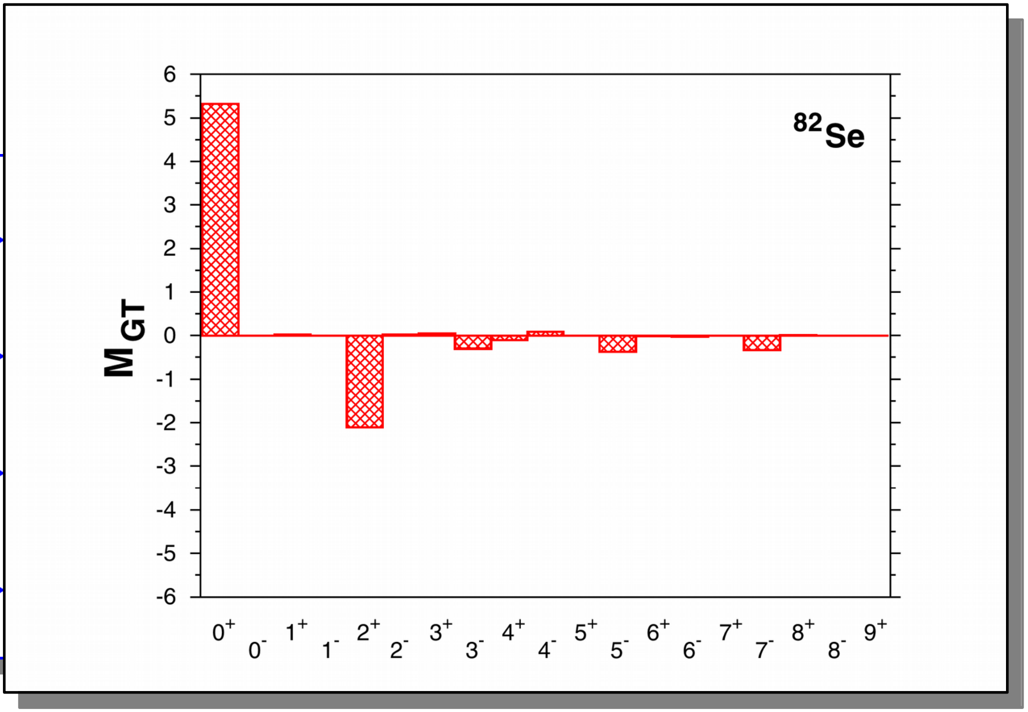
$$h_X^{F,GT,T} = \sum_{\substack{j_\pi j'_\pi \\ j_\nu j'_\nu \\ J}} -\frac{1}{4}(-1)^J\, G_X^{F,GT,T}\,(j_\pi j'_\pi j_\nu j'_\nu; J) \times \sqrt{1+(-1)^J \delta_{j_\pi j'_\pi}}\sqrt{1+(-1)^J \delta_{j_\nu j'_\nu}} \times \left(C^\dagger_{j_\pi} \times C^\dagger_{j'_\pi}\right)^{(J)} \cdot \left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(J)}$$

$$\begin{aligned}
\left(C^\dagger_{j_\pi} \times C^\dagger_{j_\pi}\right)^{(0)} &\longmapsto A_{j_\pi} s^\dagger_\pi + A'_{j_\pi} s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(0)} + \ldots \\
\left(C^\dagger_{j_\pi} \times C^\dagger_{j'_\pi}\right)^{(2)} &\longmapsto B_{j_\pi j'_\pi} d^\dagger_\pi + C_{j_\pi j'_\pi}\, s^\dagger_\pi \left(s^\dagger_\pi \tilde{d}_\pi\right)^{(2)} + \ldots \\
\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j_\nu}\right)^{(0)} &\longmapsto \tilde{A}_{j_\nu} \tilde{s}_\nu + \tilde{A}'_{j_\nu} \tilde{s}_\nu \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(0)} + \ldots \\
\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(2)} &\longmapsto \tilde{B}_{j_\nu j'_\nu} \tilde{d}_\nu + \tilde{C}_{j_\nu j'_\nu}\, (d^\dagger_\nu \tilde{s}_\nu)^{(2)}\, \tilde{s}_\nu + \ldots
\end{aligned}$$

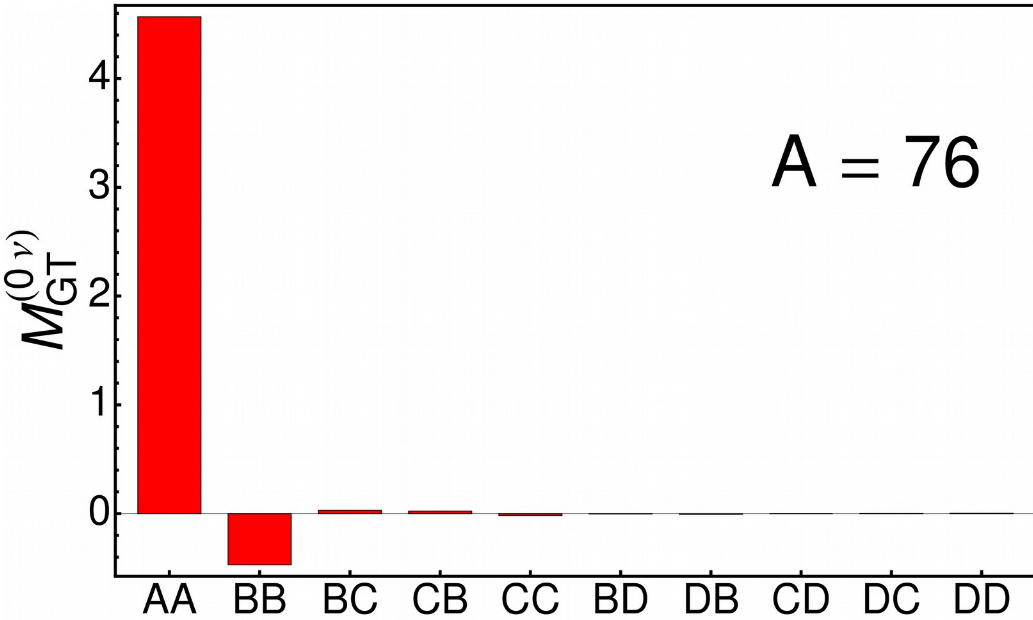
Double beta decay in the IBM-2

$$
\begin{aligned}
h_X^{F,GT,T} = \sum_{\substack{j_\pi j'_\pi \\ j_\nu j'_\nu \\ J}} & -\frac{1}{4}(-1)^J \, G_X^{F,GT,T}\,(j_\pi j'_\pi j_\nu j'_\nu; J) \\
& \times \sqrt{1+(-1)^J \delta_{j_\pi j'_\pi}}\,\sqrt{1+(-1)^J \delta_{j_\nu j'_\nu}} \\
& \times \left(C^\dagger_{j_\pi} \times C^\dagger_{j'_\pi}\right)^{(J)} \cdot \left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(J)}
\end{aligned}
$$

$$
\begin{aligned}
\left(C^\dagger_{j_\pi} \times C^\dagger_{j_\pi}\right)^{(0)} & \longmapsto \\
\left(C^\dagger_{j_\pi} \times C^\dagger_{j'_\pi}\right)^{(2)} & \longmapsto \\
\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j_\nu}\right)^{(0)} & \longmapsto \\
\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(2)} & \longmapsto
\end{aligned}
$$



Double beta decay in the IBM-2



$$ {}^{,T} \left(j_\pi j'_\pi j_\nu j'_\nu ; J\right) $$

$$ \sqrt{1+(-1)^J \delta_{j_\nu j'_\nu}} $$

$$ \cdot \left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(J)} $$

$$
\begin{aligned}
\left(C^\dagger_{j_\pi} \times C^\dagger_{j_\pi}\right)^{(0)} &\longmapsto A_{j_\pi} s^\dagger_\pi + A'_{j_\pi} s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(0)} + \ldots \\
\left(C^\dagger_{j_\pi} \times C^\dagger_{j'_\pi}\right)^{(2)} &\longmapsto B_{j_\pi j'_\pi} d^\dagger_\pi + C_{j_\pi j'_\pi} s^\dagger_\pi \left(s^\dagger_\pi \tilde{d}_\pi\right)^{(2)} + \ldots \\
\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j_\nu}\right)^{(0)} &\longmapsto \tilde{A}_{j_\nu} \tilde{s}_\nu + \tilde{A}'_{j_\nu} \tilde{s}_\nu \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(0)} + \ldots \\
\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(2)} &\longmapsto \tilde{B}_{j_\nu j'_\nu} \tilde{d}_\nu + \tilde{C}_{j_\nu j'_\nu} \left(d^\dagger_\nu \tilde{s}_\nu\right)^{(2)} \tilde{s}_\nu + \ldots
\end{aligned}
$$

Double beta decay in the IBM-2

$$h_X^{F,GT,T} = \sum_{\substack{j_\pi j'_\pi \\ j_\nu j'_\nu \\ J}} -\frac{1}{4}(-1)^J G_X^{F,GT,T}(j_\pi j'_\pi j_\nu j'_\nu; J) \times \sqrt{1+(-1)^J \delta_{j_\pi j'_\pi}} \sqrt{1+(-1)^J \delta_{j_\nu j'_\nu}} \times \left(C^\dagger_{j_\pi} \times C^\dagger_{j'_\pi}\right)^{(J)} \cdot \left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(J)}$$

$$\begin{aligned}
\left(C^\dagger_{j_\pi} \times C^\dagger_{j_\pi}\right)^{(0)} &\longmapsto A_{j_\pi} s^\dagger_\pi + A'_{j_\pi} s^\dagger_\pi \left(d^\dagger_\pi \tilde{d}_\pi\right)^{(0)} + \ldots \\
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\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j_\nu}\right)^{(0)} &\longmapsto \tilde{A}_{j_\nu} \tilde{s}_\nu + \tilde{A}'_{j_\nu} \tilde{s}_\nu \left(d^\dagger_\nu \tilde{d}_\nu\right)^{(0)} + \ldots \\
\left(\tilde{C}_{j_\nu} \times \tilde{C}_{j'_\nu}\right)^{(2)} &\longmapsto \tilde{B}_{j_\nu j'_\nu} \tilde{d}_\nu + \tilde{C}_{j_\nu j'_\nu} \left(d^\dagger_\nu \tilde{s}_\nu\right)^{(2)} \tilde{s}_\nu + \ldots
\end{aligned}$$

IBM-2 DBD calculation

Single Particle Energies

S.D.I Calculation

Structure Coefficients of S and D pairs

Mapping coefficients of the transition operator

Experimental data: spectra, EM transitions, ...

IBM-2 wavefunctions

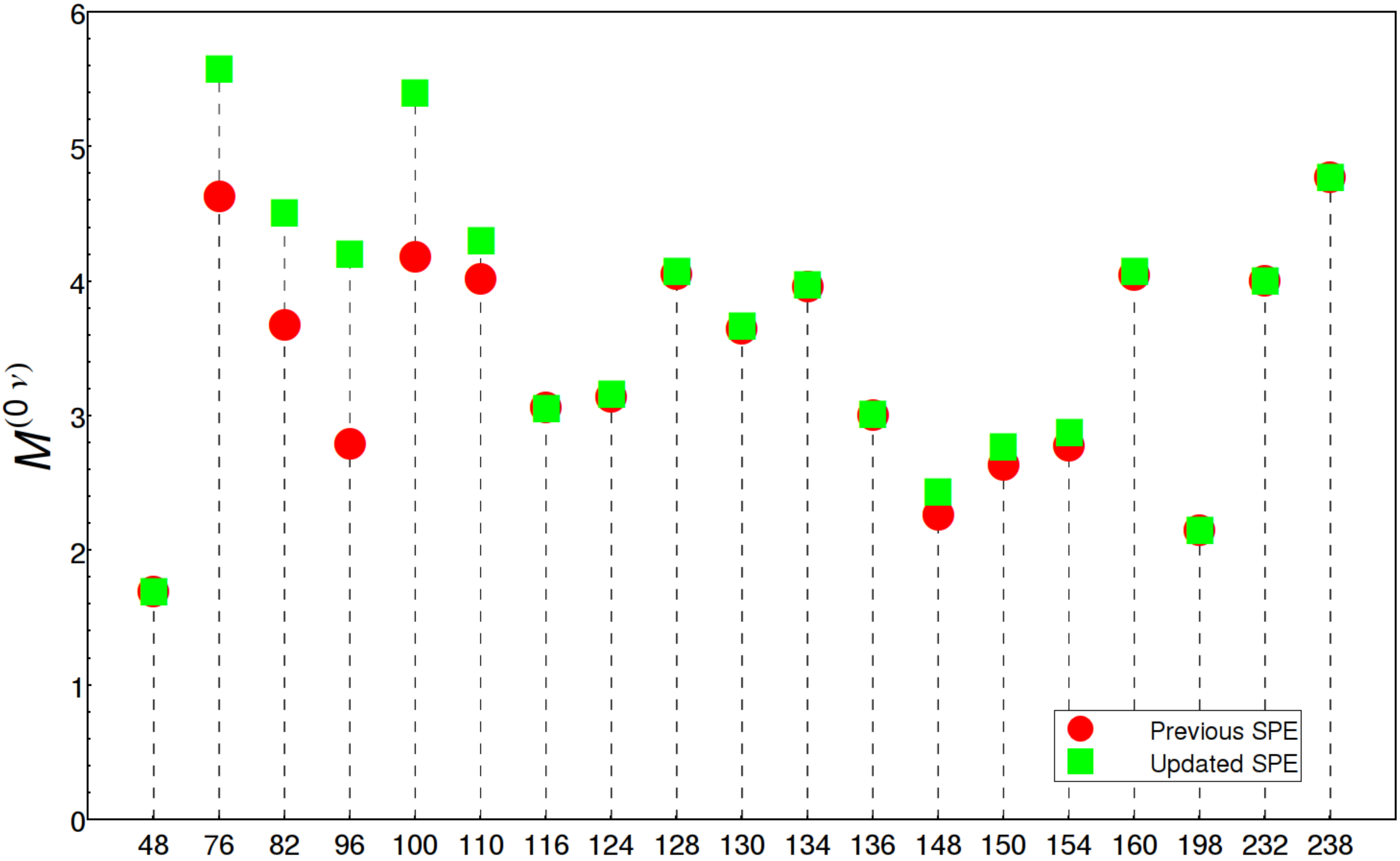
Bosonic matrix elements:

$$\left\langle F \left| s_\pi^\dagger \cdot \tilde{s}_\nu \right| I \right\rangle$$

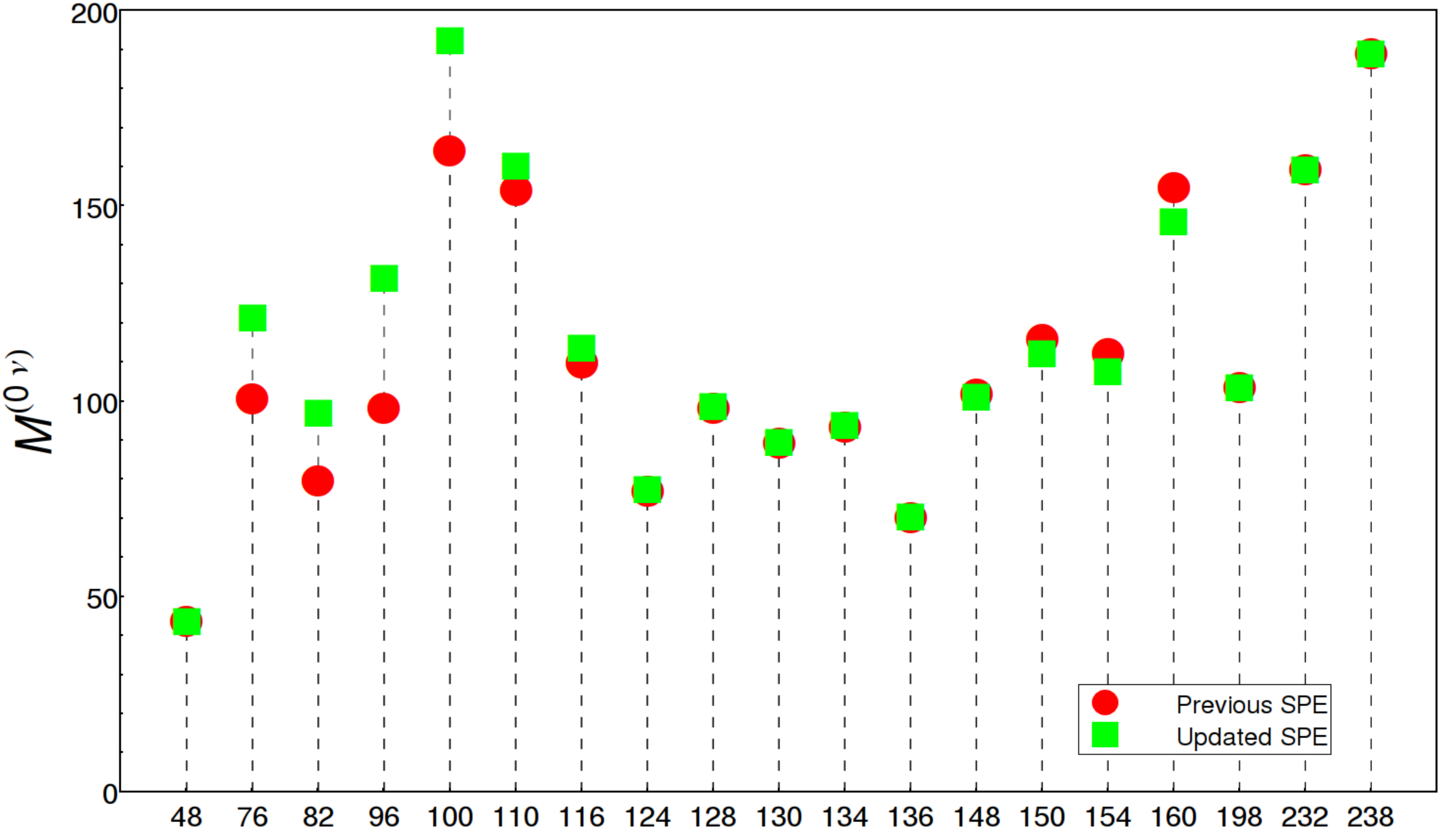
$$\left\langle F \left| d_\pi^\dagger \cdot \tilde{d}_\nu \right| I \right\rangle$$

$$\boxed{\left[\tau_{1/2}^{(0\nu)}\right]^{-1} \simeq G_{0\nu} \left| M_{0\nu} \right|^2 \left| f(m_i, U_{ei}) \right|^2}$$

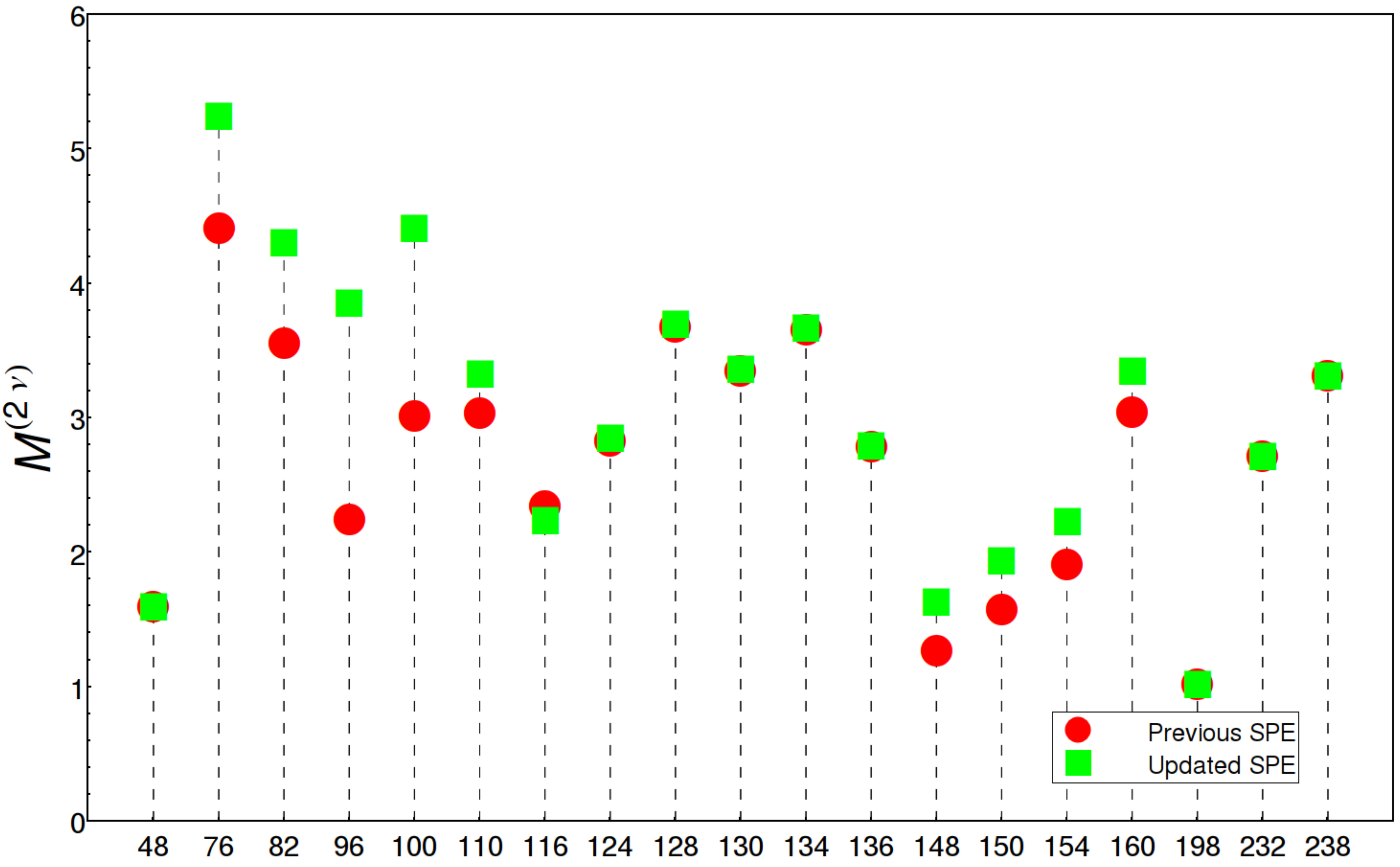
Neutrinoless DBD, light ν



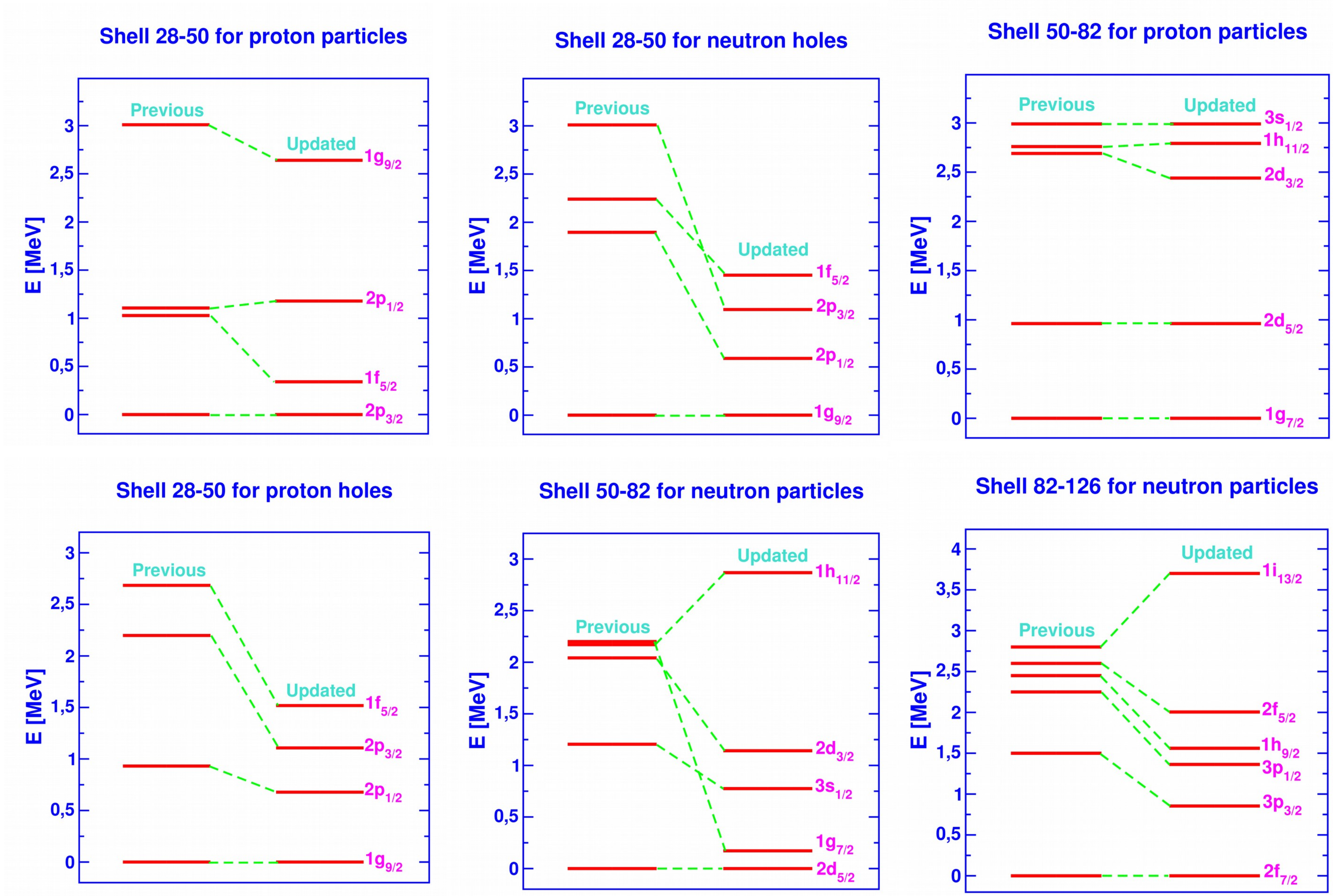
Neutrinoless DBD, heavy ν



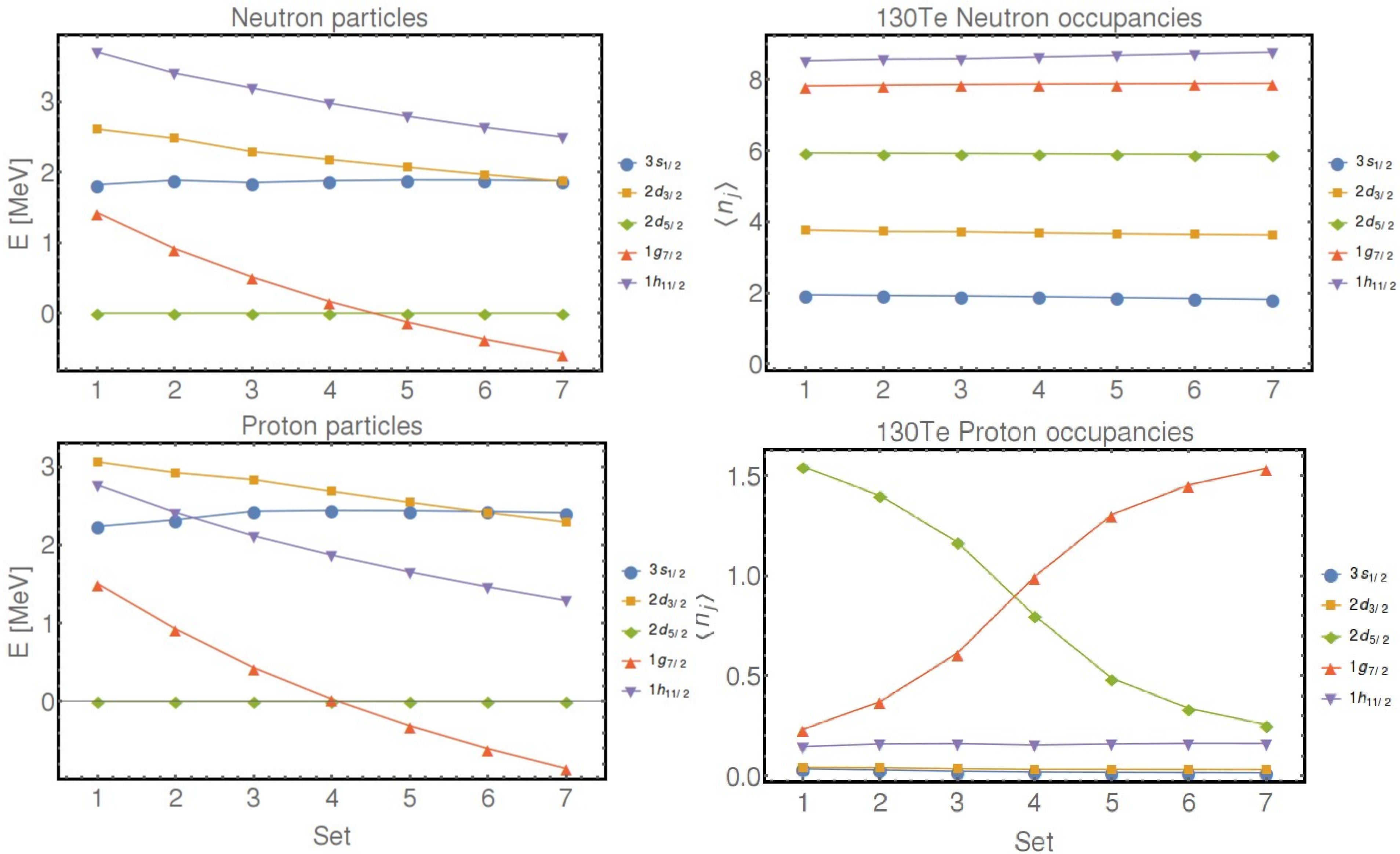
Two-neutrino DBD



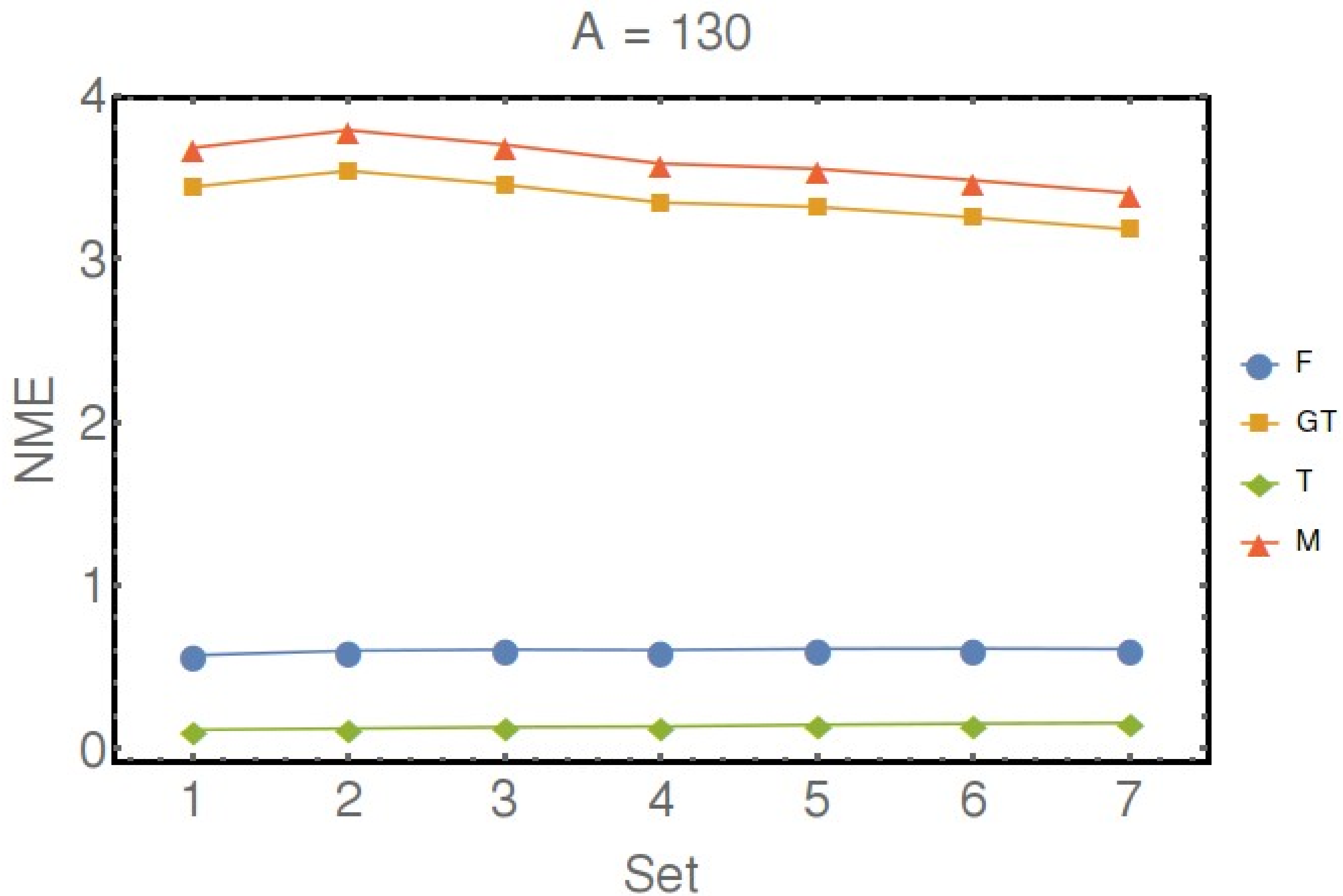
Changes in the S.P.E.



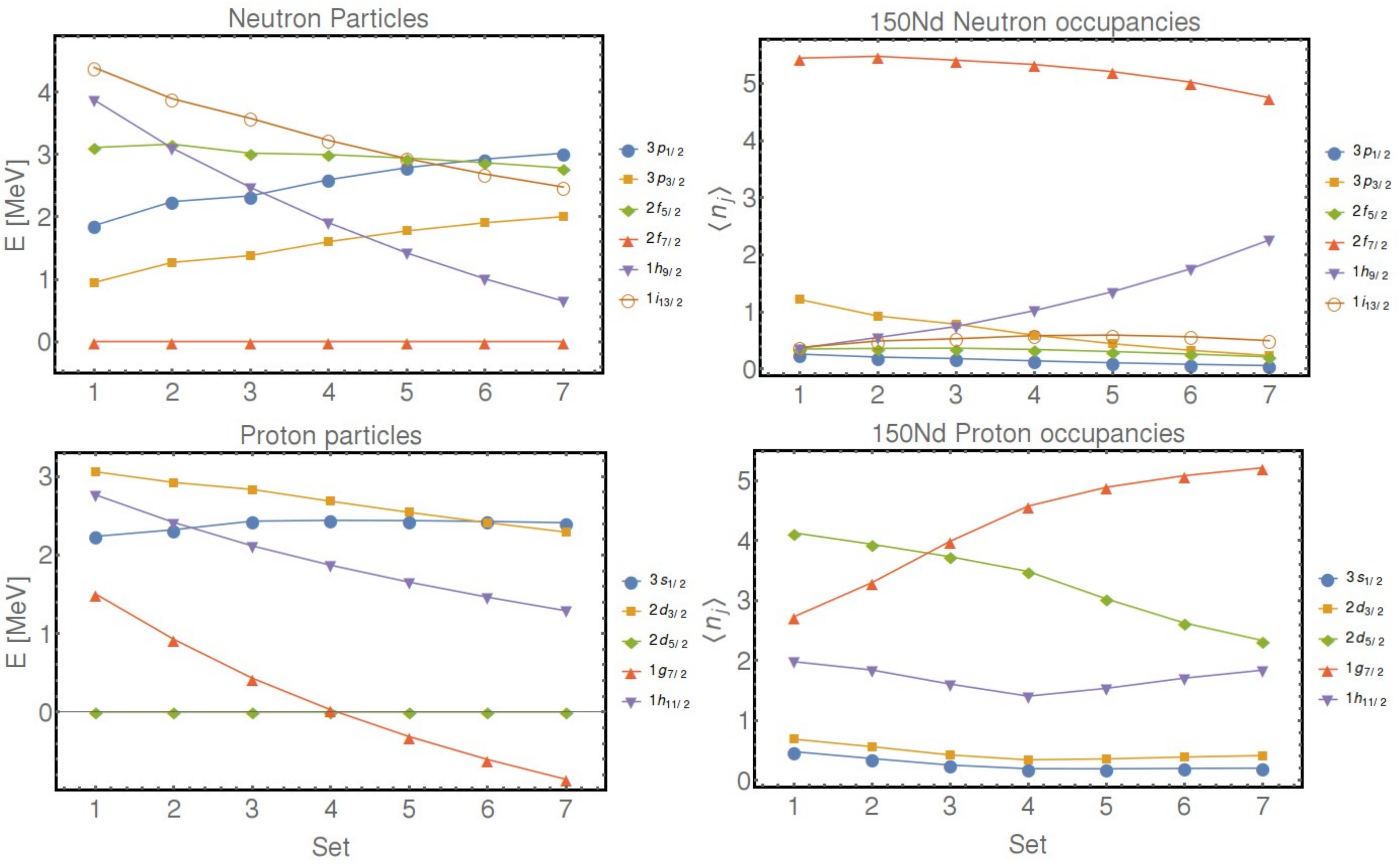
Occupations for ^{130}Te



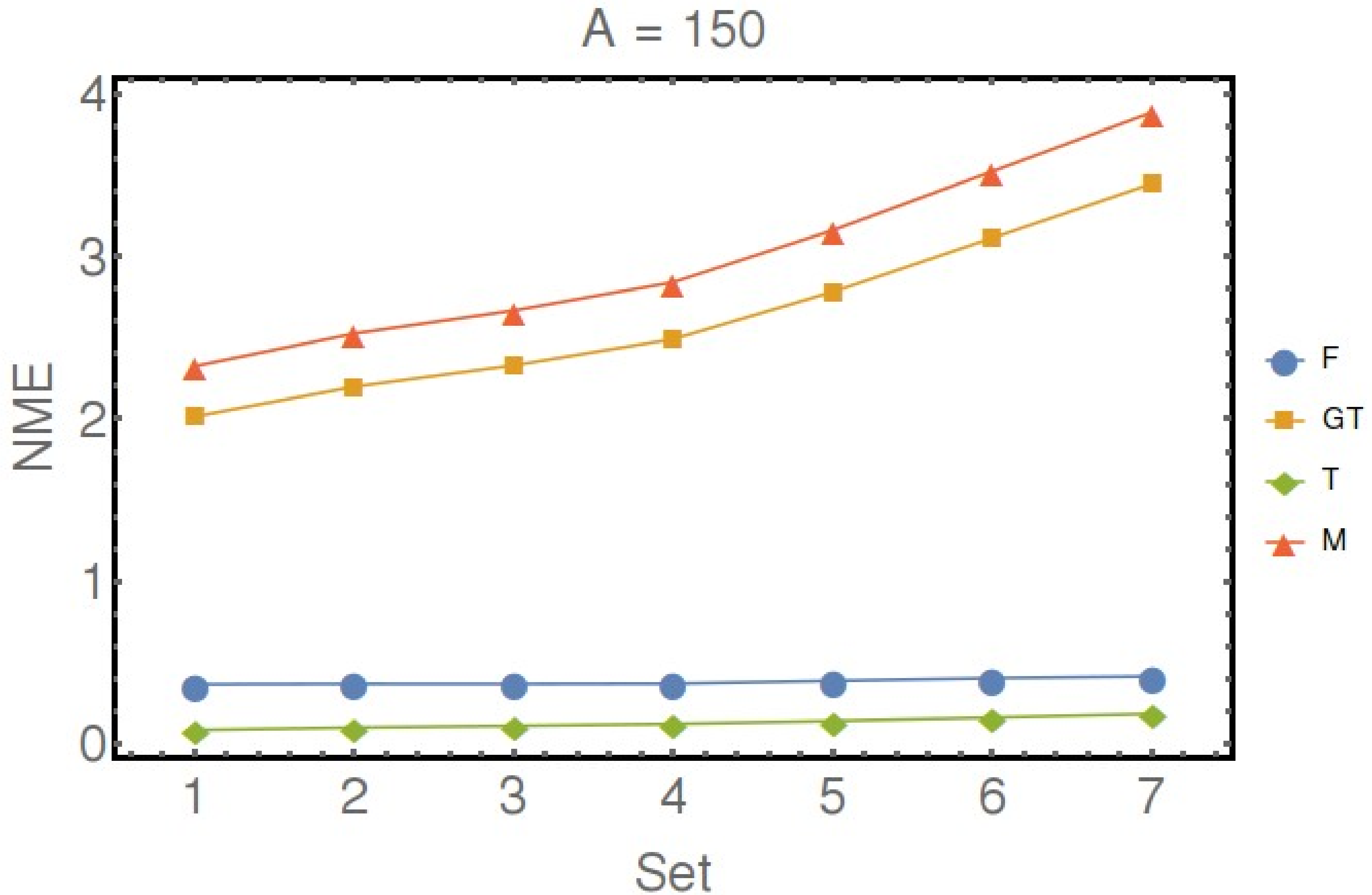
Evolution of the NME for A = 130



Occupations for ^{150}Nd



Evolution of the NME for A = 150



Summary

- Occupancies calculated with updated single particle energies reproduce data reasonably.
- DBD NMEs increase with the updated single particle energies for some cases.
- The increase is enhanced when nuclei move away from closed shells.

Thanks

Effects of the mapping coefficients

$$
\begin{aligned}
B_{jj'} &= -\sqrt{5\left(1+\delta_{jj'}\right)}\,\frac{\eta^2_{n+2,2,2}(j'j)}{\eta_{n,0,0}\,\eta_{n+2,2,2}}\,\beta_{j'j} \\
\eta^2_{n,0,0} &= \left(\frac{n}{2}!\right)^2 \sum_{m_1\ldots m_k}\left\{\prod_{i=1}^{k}\alpha_{j_i}^{2m_i}\begin{pmatrix}\Omega_{j_i}\\ m_i\end{pmatrix}\right\} \\
\eta^2_{n,2,2} &= \sum_{j\le j'}\beta^2_{jj'}\,\eta^2_{n,2,2}(jj') \\
\eta^2_{n,2,2}(jj') &= \sum_{p=0}^{\frac{n}{2}-1}\left[\frac{\left(\frac{n}{2}-1\right)!}{p!}\right]^2 (-1)^{\frac{n}{2}-1-p}\,\eta^2_{2p,0,0} \\
&\times \sum_{q=0}^{\frac{n}{2}-1-p}\alpha_j^{n-2-2p-2q}\,\alpha_{j'}^{2q}
\end{aligned}
$$